State–Space Parameter Identification in a Second Control Laboratory

J. C. Basilio and M. V. Moreira

Abstract—**A difficulty usually encountered in the preparation of a state–space-oriented control laboratory is that most of the system identification techniques available in the literature are for input/output models. Although in recent years there has been a growing interest in state–space identification methods, the application of these techniques in undergraduate courses are not immediate since they require a deep knowledge in mathematics and system theory. In this paper, experiments are proposed to estimate the parameters of a second-order state–space system, a dc motor-generator group, whose model plays a key role in a laboratory that deals with state–space design. The efficiency of the proposed experiments is demonstrated with the estimation of all parameters of a real system.**

Index Terms—**Control education, laboratory education, parameter estimation, state–space models.**

I. Introduction

IN GENERAL, laboratories for the basic courses in control systems are divided as follows. First, an initial one [1] uses, in general, step- and frequency-response methods to obtain a transfer function for the system and, in the sequel, by deploying classical control tools [2], proportional+integral (PI) or proportional+integral+derivative (PID) compensators are designed to meet transient performance specifications. Second, a state–space-based control laboratory should also cover the following topics: modeling/identification, controller design, simulation, and controller implementation. From the didactic point of view, ideal plants for a state–space-oriented laboratory are those whose states are available. This feature allows the following topics to be covered [3]: state feedback, state estimators, and design for robust tracking and disturbance rejection. In addition, it is important that the plant has, at least, two states, i.e., be modeled as a second-order system.

A difficulty usually encountered in the preparation of a state–space-oriented control laboratory is that most of the system identification techniques available in the literature [4]–[6] are for input–output models. Although in recent years there has been a growing interest in state–space identification methods [7]–[10] and references therein, the application of these techniques in undergraduate courses are not immediate since they require a deep knowledge in mathematics and system theory. In this paper, experiments are proposed to estimate the parameters of a second-order state–space system, a dc motor-generator group, whose model plays a key role in a laboratory that deals with state–space design. The key point of the proposed identification scheme is to split the second-order system into two first-order ones and, in the sequel, to obtain the equivalent discrete-time model for the first-order models. The parameters can then be estimated using least squares.

This paper is structured as follows. In Section II, a discrete-time model equivalent to a continuous first-order model will be reviewed, and general experiments for the estimation of the parameters of these systems are proposed. In Section III, a second-order state–space model for the motor-generator group is initially obtained, and in Section IV, experiments are proposed to carry out the parameter identifications in a control laboratory. The efficiency of the proposed experiments will be demonstrated in Section V with the estimation of all parameters of a dc motor-generator group of the Control Laboratory of the Federal University of Rio de Janeiro (UFRJ), Rio de Janeiro, Brazil. Finally, conclusions are drawn in Section VI.

II. A Discrete-Time Model for a First-Order Continuous-Time System

The experiments to be proposed in this paper for the identification of the plant parameters are based on the estimation of the parameters of a first-order continuous-time system. However, as explained in the next section, frequency- or step-response methods cannot be deployed, since sine-type or step signals cannot be applied directly to the resulting first-order systems. This apparent difficulty is circumvented using parametric identification. Thus, an equivalent discrete-time model for the first-order continuous-time system needs to be obtained. Since this relationship is crucial in the understanding of the proposed identification scheme, the topics considered in this section should receive special attention in the theoretical course which precedes the laboratory or in the preparatory part of a control laboratory that deploys the methodology proposed here.

A. Background

The transfer function of a first-order system with time constant τ and steady-state gain K, with no time delay, is given by

$$G(s) = \frac{Y(s)}{U(s)} = \frac{K}{\tau s + 1}. \tag{1}$$

A straightforward state–space representation for (1) is

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \tag{2}$$

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where $A = -1/\tau$, $B = K/\tau$ e $C = 1$. It is well known from control theory [11] that the discrete-time model equivalent to a continuous-time system with the state–space description (2) is given by

$$\begin{cases} x(t_{k+1}) = \Phi x(t_k) + \Gamma u(t_k) \\ y(t_k) = Cx(t_k) \end{cases} \tag{3}$$

where t_k and t_{k+1} denote the sampling instants

$$\begin{cases} \Phi = e^{Ah} = e^{-\frac{h}{\tau}} \\ \Gamma = \int_0^h e^{Ax} B dx = K(1 - \Phi) \end{cases} \tag{4}$$

with $h = t_{k+1} - t_k$ denoting the sampling period. One can easily see that (3) leads to the linear difference equation

$$y(t_k) = \Phi y(t_{k-1}) + \Gamma u(t_{k-1}). \tag{5}$$

This expression shows that, providing the parameters Φ and Γ are estimated, then the gain K and the time constant τ of the continuous-time model (1) can be computed using the relationships given in (4).

B. Least-Squares Estimation of Φ and Γ

The determination of Φ and Γ can be carried out as follows [11, p. 423]: assume that the output $y(t_k)$ and input $u(t_k)$ are known for $k = 0, 1, \ldots, q$. Thus, from (5), one can write

$$\begin{cases} y(t_1) = \Phi y(t_0) + \Gamma u(t_0) \\ y(t_2) = \Phi y(t_1) + \Gamma u(t_1) \\ \quad \vdots \\ y(t_q) = \Phi y(t_{q-1}) + \Gamma u(t_{q-1}) \end{cases} \tag{6}$$

whose equivalent matrix form is given as

$$\underline{b} = A\underline{x} \tag{7}$$

where

$$\underline{b} = \begin{bmatrix} y(t_1) \\ y(t_2) \\ \vdots \\ y(t_q) \end{bmatrix}, \quad A = \begin{bmatrix} y(t_0) & u(t_0) \\ y(t_1) & u(t_1) \\ \vdots & \vdots \\ y(t_{q-1}) & u(t_{q-1}) \end{bmatrix}, \quad \underline{x} = \begin{bmatrix} \Phi \\ \Gamma \end{bmatrix}.$$

It is important to remark that, in practice, $q+1$ (the number of recorded inputs and outputs) is much greater than 2 (the number of columns of A). This fact makes almost impossible a solution for system (7), and, therefore, one has to seek an approximate solution. A well-known solution to this problem is the "least-squares" solution, i.e., the one which minimizes the Euclidean norm of $\underline{e} = \underline{b} - A\underline{x}$ and is given by [12]

$$\underline{x} = (A^t A)^{-1} A^t \underline{b}. \tag{8}$$

Remark A: In the least-squares estimation of discrete-time systems, the shape of the input signal plays an important role since it can make the matrix A in (7) lose rank, precluding the computation of the inverse of $A^t A$ in (8). Such a problem can be avoided by using, as input signals, those that are sufficiently rich [11, p. 423], as far as frequency information is concerned. In the present work, a pseudorandom binary signal (PRBS) will be used as input. This signal can also be seen as a pulse train with variable width [6, p. 373]. □

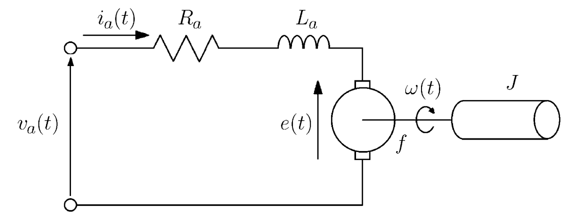

Fig. 1. Equivalent circuit of an armature controlled dc motor.

III. Modeling

The plant used in the Control Laboratory of UFRJ consists of a motor-generator group (two armature-controlled dc motors connected by their shafts). The variable to be controlled is the shaft velocity, which is usually chosen to illustrate the following concepts [13]: 1) the need for feedback and 2) the need for dynamic compensation when signals modeled as step are to be tracked or rejected. In addition, this plant is also appropriate for state–space-oriented control laboratories since, as it will be seen in this section, it can be modeled as a second-order system. Furthermore, one can always define state variables that can be accessible for measurement and state feedback.

The generator in the motor-generator group plays an important role, i.e., the effect of connecting a load (e.g., a resistance) to its terminals can be modeled as an external disturbance signal applied to the plant input [1]. The consequence is that the mathematical model of a dc motor-generator group is the same as that of an armature-controlled dc motor, whose equivalent circuit is shown in Fig. 1, where R_a and L_a denote, respectively, the armature resistance and inductance, J is the motor inertia, and f represents the viscous friction.

Remark B: In order to derive a model for this system, one must take into account two types of frictions: Coulomb (dry) and viscous frictions. The main contributors to the frictions in a dc motor are the frictions between the brushes and the commutator, as well as in the bearings (Coulomb friction), and is a result of windage (viscous friction). In well-lubricated bearings, there is a component of viscous friction because of the laminar flow of the lubricant, being usually the predominant one, when the shaft is already in movement and a component of Coulomb friction at very slow speed [14]. Therefore, since the model will be assumed to be linear, the only effect that should be taken into account is that of the viscous friction. The Coulomb friction has a significant contribution to the motor nonlinear behavior when very low voltages are applied to the motor input. This effect, usually referred in the literature to as a dead zone, will be illustrated in Section IV. □

Direct application of Kirchoff and Newton laws to the system of Fig. 1 leads to the equations

$$\begin{cases} v_a(t) = R_a i_a(t) + L_a \frac{d}{dt} i_a(t) + e(t) \\ t_m(t) - t_d(t) - f\omega(t) = J \frac{d}{dt}\omega(t) \end{cases} \tag{9}$$

where $t_m(t)$ denotes the motor torque and $t_d(t)$ an external torque, which can be neglected when no load is connected to the generator terminals. The relationships between electrical and mechanical equations are given as

$$\begin{cases} e(t) = K_g\omega(t) \\ t_m(t) = K_a i_a(t) \end{cases} \tag{10}$$

where $\omega(t)$ denotes the shaft velocity, and K_g and K_a are, respectively, the counter electromotive and torque constants. Substituting (10) in (9), and defining as state variables the armature current $i_a(t)$ and the shaft velocity $\omega(t)$, and as input, the external voltage applied to the armature circuit $v_a(t)$, the following state–space model is obtained:

$$\begin{bmatrix} \frac{d}{d} i_a(t) \\ \frac{d}{d}\omega(t) \end{bmatrix} = \begin{bmatrix} -\frac{R_a}{L_a} & -\frac{K_g}{L_a} \\ \frac{K_a}{J} & -\frac{f}{J} \end{bmatrix} \begin{bmatrix} i_a(t) \\ \omega(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{L_a} \\ 0 \end{bmatrix} v_a(t). \tag{11}$$

Therefore, for the complete description of a second-order state–space model of an armature-controlled dc motor, it is necessary to identify the parameters R_a, L_a, J, f, K_g, and K_a. The identification of R_a and L_a can be carried out using concepts of electric machines [15]. However, most of electric-machine textbooks present models that are valid only for a steady-state regime, thus neglecting the effect of the armature inductance. The inertia J can be determined using concepts from Mechanics, which usually requires the motor to be disassembled. Another important point to remark is that the viscous friction f varies linearly with the shaft velocity; therefore, one needs to obtain an average value for the viscous friction in the linear region of operation. Hence, one must seek alternative ways to identify R_a and L_a and to design an experiment that takes into account the different operating conditions of the motor. One should emphasize that the experiments proposed in this paper require only basic knowledge of electric machines, usually presented in standard control textbooks (see, e.g., [2]).

Finally, notice that the measurement of $\omega(t)$ is usually performed by encoders or tachometers. These elements generate a voltage at their terminals $(v_t(t))$ proportional to $\omega(t)$, as follows:

$$v_t(t) = K_t\omega(t). \tag{12}$$

Thus, substituting (12) in (11), results in

$$\begin{bmatrix} \frac{d}{d} i_a(t) \\ \frac{d}{d} v_t(t) \end{bmatrix} = \begin{bmatrix} -\frac{R_a}{L_a} & -\frac{K_g}{K_t L_a} \\ \frac{K_a K_t}{J} & -\frac{f}{J} \end{bmatrix} \begin{bmatrix} i_a(t) \\ v_t(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{L_a} \\ 0 \end{bmatrix} v_a(t). \tag{13}$$

IV. Parameter Estimation

In order to design experiments for the estimation of R_a, L_a, J, f, and K_a, notice that (13) can also be written as

$$\begin{cases} L_a \frac{d}{dt} i_a(t) + R_a i_a(t) = v_a(t) - \frac{K_g}{K_t} v_t(t) \\ J\frac{d}{dt} v_t(t) + f v_t(t) = K_a K_t i_a(t). \end{cases} \tag{14}$$

Thus, defining

$$\begin{cases} u_e(t) = v_a(t) - \frac{K_g}{K_t} v_t(t) \\ u_m(t) = K_a K_t i_a(t) \end{cases} \tag{15}$$

then (14) is equivalent to

$$\begin{cases} L_a \frac{d}{dt} i_a(t) + R_a i_a(t) = u_e(t) \\ J\frac{d}{dt} v_t(t) + f v_t(t) = u_m(t) \end{cases} \tag{16}$$

which represents two first-order systems. This finding allows one to define two systems, an electrical (E) and a mechanical (M), whose state–space realizations are given as

$$(E)\begin{cases} \dot{x}_e(t) = A_e x_e(t) + B_e u_e(t) \\ y_e(t) = x_e(t) \end{cases} \tag{17}$$

where $x_e(t) = i_a(t)$, $A_e = -R_a/L_a$ and $B_e = 1/L_a$ and

$$(M)\begin{cases} \dot{x}_m(t) = A_m x_m(t) + B_m u_m(t) \\ y_m(t) = x_m(t) \end{cases} \tag{18}$$

where $x_m(t) = v_t(t)$, $A_m = -f/J$ and $B_m = 1/J$.

According to (15), input $u_e(t)$ depends on a control variable $v_a(t)$, and another one $v_t(t)$ depends on the machine parameters. Similar conclusions could be drawn for $u_m(t)$, which depends on $i_a(t)$ being, therefore, a function of the motor parameters. An immediate consequence of this fact is that the identification of R_a, L_a, J, and f cannot be carried out by continuous-time identification techniques, such as frequency or step response. This apparent problem can be circumvented if the discrete-time system identification technique suggested in the previous section is deployed.

Since the plant is being described by the linear model given in (13), its linear region of operation must be determined. An experiment for the determination of the linear region of operation of the plant should, therefore, be the first experiment to be performed.

A. Determination of the Linear Region of Operation

Experiments for the determination of a linear region of operation, as well as the kind of nonlinearity present in a dc motor, has already been considered in [1]. Here, only the algorithms necessary to determine the input values for which the system behaves as a linear one are presented. A more detailed explanation can be found in [1]. In addition, it will be assumed here that the tachometer gain K_t is known. If K_t is unknown, then its value can be determined by least-squares fitting of the points (ω_i, V_{t_i}), $i = 1, 2, \ldots, n$, where ω_i is the shaft velocity and V_{t_i} is the corresponding voltage at the tachometer terminals.

Let V_a denote the step amplitude of a voltage signal applied to the armature circuit of the dc motor, and V_t denote the corresponding steady-state value of the voltage at the tachometer terminals. The determination of the linear region of operation can be carried out as follows.

Experiment 1:

Step 1) Apply to the armature circuit step signals of amplitude V_{a_i}, $i = 1, 2, \ldots, n$, and record the corresponding steady-state values of the voltage at the tachometer terminals V_{t_i}, $i = 1, 2, \ldots, n$.

Step 2) Form the points (V_{a_i}, V_{t_i}), $i = 1, \ldots, n$, and find the coefficients a_k, $k = 0, 1, \ldots, q$ of a polynomial $V_t = p(V_a)$ of degree q, i.e.,

$$V_t = a_0 V_a^q + a_1 V_a^{q-1} + \cdots + a_{q-1} V_a + a_q \tag{19}$$

which fits better (in a least-squares sense) to the points (V_{a_i}, V_{t_i}). □

The coefficients $a_i, i = 0, 1, \ldots, q$ of the polynomial $V_t = p(V_a)$, given in (19), can be computed by least-squares fitting. Notice that, with the points obtained from the experiment, the following system of equations can be formed:

$$\begin{bmatrix} V_{t_1} \\ V_{t_2} \\ \vdots \\ V_{t_n} \end{bmatrix} = \begin{bmatrix} V_{a_1}^q & V_{a_1}^{q-1} & \cdots & V_{a_1} \\ V_{a_2}^q & V_{a_2}^{q-1} & \cdots & V_{a_2} \\ \vdots & \vdots & \vdots & \vdots \\ V_{a_n}^q & V_{a_n}^{q-1} & \cdots & V_{a_n} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_{q-1} \end{bmatrix} \tag{20}$$

which can be written as $\underline{b} = A\underline{x}$. Since the number of rows (number of points recorded in the experiment) is usually much greater than the number of columns of A (the degree of the polynomial $V_t = p(V_a)$ plus one), once again, one must rely on a least-squares solution to compute the coefficients $a_k, k = 0, 1, \ldots, q$. One can easily see that [12] $\underline{x} = [a_0\ a_1\ \cdots\ a_q]^t = (A^tA)^{-1}A^t\underline{b}$ is the desired solution. Once the polynomial $V_t = p(V_a)$ has been computed, the linear region of operation will be given by the values of V_a for which the derivative of V_t with respect to V_a is approximately constant.

Remark C:

1) One way to determine q is as follows: for q $= 1$, compute $\|\underline{e}\|_2 = \|A\underline{x} - \underline{b}\|_2$. Then, increase q until the matrix A becomes nearly singular[1] or the decrease in $\|\underline{e}\|_2$ is below a prescribed value.
2) Besides being a systematic way to determine the linear region of operation, the experiment proposed above also allows the student to identify the type of nonlinearity present in the process. In the case of dc motors, for steps with amplitude below a certain value V_0, the motor does not rotate. As mentioned in remark B, this situation characterizes a nonlinear effect called dead zone. □

B. Computation of K_g and K_a

When expressed in international system units, the electromotive and torque constants K_g and K_a, respectively, have the same figures. Therefore, only one experiment to find them is sufficient. In addition, notice from (10) that, if a constant external torque is applied to the shaft, i.e., the dc motor is made to work as a generator, the steady-state-induced voltage at its armature terminal (with no load connected) will be given by:

$$E = \left(\frac{K_g}{K_t}\right) V_t. \tag{21}$$

This expression suggests the following experiment for the computation of K_g.

Experiment 2:

Step 1) Apply an external torque to the motor and record the voltage at the tachometer terminals (V_{t_i}) and the induced voltage at the armature terminals (V_{a_i}) for different speeds. Since there is no load connected to the armature circuit, then V_{a_i} is approximately equal E_i.

Step 2) Form the vectors

$$\underline{e} = [E_1 \quad E_2 \quad \cdots \quad E_n]^t \text{ and } \underline{v} = [V_{t_1} \quad V_{t_2} \quad \cdots \quad V_{t_n}]^t.$$

Step 3) Compute K_g as follows:

$$K_g = K_t \frac{\underline{v}^t \underline{e}}{\underline{v}^t \underline{v}}. \tag{22}$$

□

Remark D: If the shaft velocity (W_i) is recorded, instead of the voltage at the tachometer terminals (V_{t_i}), one must form the vector $\underline{\omega} = [W_1\ W_2\ \cdots\ W_n]^t$, instead of $\underline{v}$, and compute K_g as follows:

$$K_g = \frac{\underline{\omega}^t \underline{e}}{\underline{\omega}^t \underline{\omega}}. \tag{23}$$

□

C. Computation of R_a, L_a, J, and f

Systems E and M, given in (17) and (18), respectively, when expressed in the transfer function form (1) are such that $K_e = 1/R_a$, $\tau_e = L_a/R_a$, $K_m = 1/f$, and $\tau_m = J/f$. Therefore, according to (3) and (4), the discretized models of systems E and M are, respectively

$$(E)\begin{cases} x_e(t_{k+1}) = \Phi_e x_e(t_k) + \Gamma_e u_e(t_k) \\ y_e(t_k) = x_e(t_k) \end{cases} \tag{24}$$

where $\Phi_e = e^{-h/\tau_e}$ and $\Gamma_e = K_e(1 - \Phi_e)$, and

$$(M)\begin{cases} x_m(t_{k+1}) = \Phi_m x_m(t_k) + \Gamma_m u_m(t_k) \\ y_m(t_k) = x_m(t_k) \end{cases} \tag{25}$$

where $\Phi_m = e^{-h/\tau_m}$ and $\Gamma_m = K_m(1 - \Phi_m)$. Consequently, linear difference equations for systems E and M will be given by

$$(E) \quad y_e(t_k) = \Phi_e y_e(t_{k-1}) + \Gamma_e u_e(t_{k-1}) \tag{26}$$

$$(M) \quad y_m(t_k) = \Phi_m y_m(t_{k-1}) + \Gamma_m u_m(t_{k-1}). \tag{27}$$

Equations (26) and (27) show that, if the parameters Φ_e, Φ_m, Γ_e, and Γ_m are determined, then the gains K_e, K_m (and, consequently, $R_a = 1/K_e$ and $f = 1/K_m$, respectively), and the time constants τ_e and τ_m (and, analogously, $L_a = R_a\tau_e$ and $J = f\tau_m$) can be immediately computed. In addition, notice that, since $y_e(t_k) = i_a(t_k)$, $y_m(t_k) = v_t(t_k)$, $u_e(t_k) = v_a(t_k) - (K_g/K_t)v_t(t_k)$, and $u_m(t_k) = K_aK_ti_a(t_k)$; then, by applying a voltage signal $v_a(t)$ according to remark A, and recording $v_a(t_k)$, $i_a(t_k)$ and $v_t(t_k)$ at the sampling instants t_k, $k = 0, 1, \ldots, n$ $(n \gg 2)$, one may write

$$\begin{bmatrix} i_a(t_1) \\ i_a(t_2) \\ i_a(t_3) \\ \vdots \\ i_a(t_n) \end{bmatrix} = \begin{bmatrix} i_a(t_0) & u_e(t_0) \\ i_a(t_1) & u_e(t_1) \\ i_a(t_2) & u_e(t_2) \\ \vdots & \vdots \\ i_a(t_{n-1}) & u_e(t_{n-1}) \end{bmatrix} \begin{bmatrix} \Phi_e \\ \Gamma_e \end{bmatrix} \Leftrightarrow \underline{i}_a = M_e \underline{x}_e \tag{28}$$

[1]From the computation point of view, a matrix can be said to be singular when these two conditions hold: 1) its smallest singular value is bellow a prescribed tolerance value, and 2) its condition number is above a given upper bound.

and

$$\begin{bmatrix} v_t(t_1) \\ v_t(t_2) \\ v_t(t_3) \\ \vdots \\ v_t(t_n) \end{bmatrix} = \begin{bmatrix} v_t(t_0) & u_m(t_0) \\ v_t(t_1) & u_m(t_1) \\ v_t(t_2) & u_m(t_2) \\ \vdots & \vdots \\ v_t(t_{n-1}) & u_m(t_{n-1}) \end{bmatrix} \begin{bmatrix} \Phi_m \\ \Gamma_m \end{bmatrix} \Leftrightarrow \underline{v}_t = M_m \underline{x}_m. \tag{29}$$

The least-squares solutions to (28) and (29) are, respectively, given by

$$\underline{x}_e = \left(M_e^t M_e\right)^{-1} M_e^t \underline{i}_a \text{ and } \underline{x}_m = \left(M_m^t M_m\right)^{-1} M_m^t \underline{v}_t. \tag{30}$$

Therefore, the motor parameters R_a, L_a, f, and J will be given by

$$R_a = \frac{1-\Phi_e}{\Gamma_e},\ L_a = -\frac{R_a h}{\ln(\Phi_e)},\ f = \frac{1-\Phi_m}{\Gamma_m},\ J = -\frac{fh}{\ln(\Phi_m)} \tag{31}$$

where h is the sampling period.

The development above leads to the following experiment to find the parameters R_a, L_a, J, and f of a dc motor.

Experiment 3:

Step 1) Apply a voltage signal at the armature terminals and record the signals $v_a(t)$, $i_a(t)$ (armature-applied voltage and armature current, respectively), and $v_t(t)$ (tachometer voltage) at the sampling instants $t = t_k$, $k = 0, 1, \ldots, n$.

Step 2) Compute $u_e(t_k) = v_a(t_k) - (K_g/K_t)v_t(t_k)$ and $u_m(t_k) = K_a K_t i_a(t_k)$, and form the vectors $\underline{i}_a$ and $\underline{v}_t$ and the matrices M_e and M_m, as defined in (28) and (29).

Step 3) Compute Φ_e, Φ_m, Γ_e, and Γ_m according to (30).

Step 4) Use expressions (31) to compute R_a, L_a, J, and f. □

Remark E:

1) The voltage signal to be applied to the armature terminal should be a pulse train with variable pulsewidth, which, according to remark A, can be seen as a PRBS. Furthermore, since the motor parameters are determined for a linear model, the pulse amplitude should be such that the input signal remains in the linear region of operation determined according to experiment 1, i.e., the minimum (maximum) value of the voltage applied to the armature terminals must be greater (smaller) than the left (right) extremes of the interval that characterizes the linear region.

2) Notice that $u_e(t_k)$ cannot be zero for all t_k. Because the motor has a monotonically increasing response, it precludes $u_e(t_k)$ from being zero during, at least, the time interval corresponding to the transient response. □

V. Practical Results

A. Experiments

In this section, the experiments proposed in the paper will be used to obtain a state–space model for a dc motor-generator group. The experiments have been carried out by the students as part of the second laboratory in Control System at UFRJ.

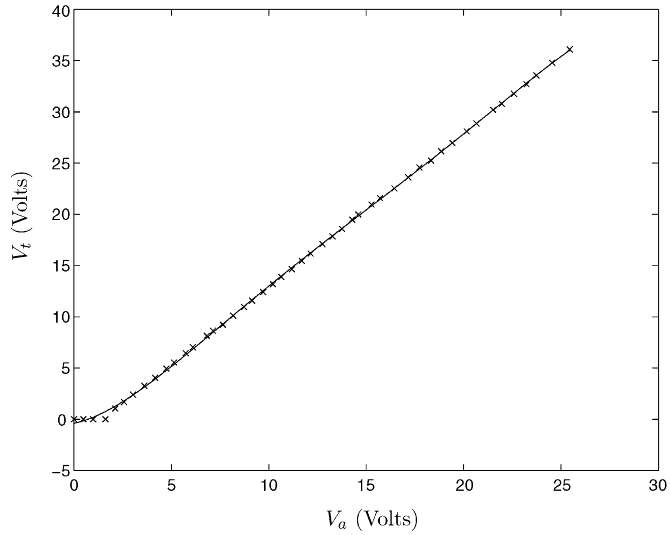

Fig. 2. Points (V_{a_i}, V_{t_i}) (x marks) and the values of $p(V_a)$ (solid line) for a fifth-order polynomial used in the determination of the linear region of the dc motor.

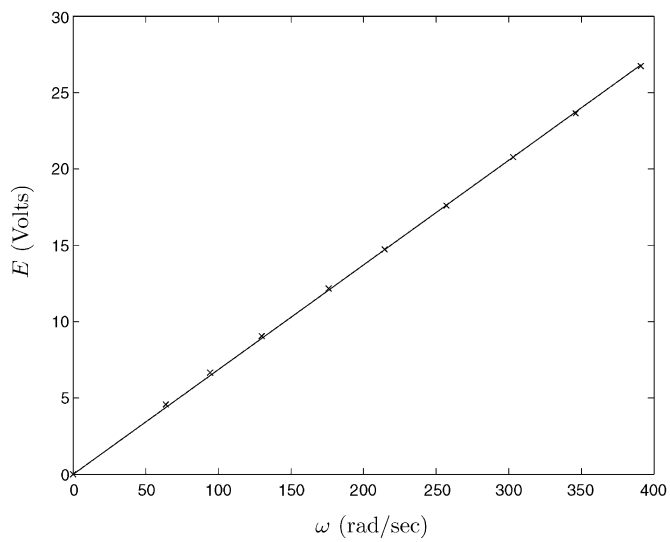

Fig. 3. Points (W_i, E_i) (x-marks) and the straight line obtained by least-squares fit.

According to Section IV, the first experiment is the determination of the linear region. As explained in remark E, this experiment defines the smallest and the largest values (and, therefore, the amplitude) of the pulse train to be used in experiment 3 to determine R_a, L_a, J, and f. Following the steps of experiment 1, the points (V_{a_i}, V_{t_i}), shown in Fig. 2 (x-marks), are obtained. These points have been fitted by a fifth-order polynomial $p(V_a)$, whose plot is also given in Fig. 2. Note that, for steps with amplitude smaller than 1.0857 V, the output is zero, i.e. the motor shaft remains stuck. As pointed out in remarks B and C, this situation reveals the existence of a dead zone. In addition, notice that the dc motor behaves as a linear system for input voltages in the interval [2,25] V.

The second experiment to be carried out is to identify K_a and K_g. Proceeding according to experiment 2, the pairs

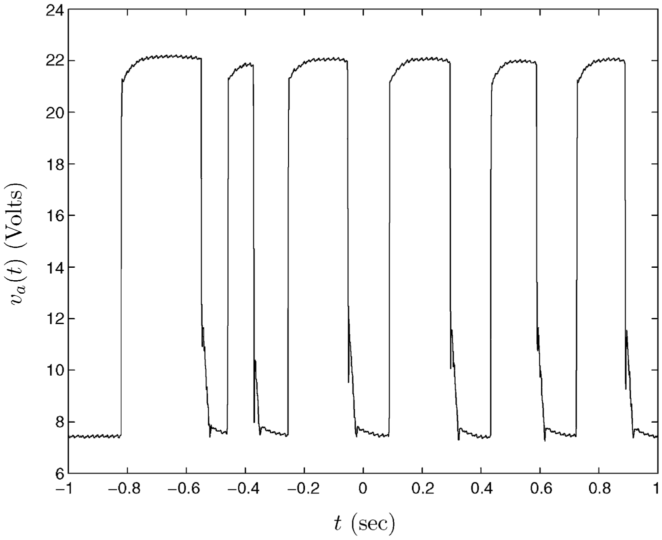

Fig. 4. Signal applied to the dc motor for the estimation of R_a, L_a, J, and f.

(W_i, E_i), plotted in Fig. 3 (x-marks), are obtained. Therefore, in accordance with Step 3, $K_g = 0.0453$ V/(rad/s) and $K_a = 0.0453$ Nm/A. The accuracy of the result can be checked by comparing the points (W_i, E_i) and the straight line $K_g\omega(t)$ in Fig. 3 (solid line).

Finally, according to experiment 3, for the identification of R_a, L_a, J, and f, one must first record the signals $v_a(t)$, $i_a(t)$, and $v_t(t)$. The signal applied to the armature terminal $v_a(t)$ is shown in Fig. 4, while the resulting signals $i_a(t)$ and $v_t(t)$ are given in Fig. 5(a) and (b), respectively, with solid lines. Following the next steps of experiment 3, the values obtained for the electrical parameters R_a and L_a are 2.30 Ω and 3.4 mH, respectively, and for the mechanical parameters J and f are $3.72 \times 10^{-5}\ \text{kg} \cdot \text{m}^2$ and $5.23 \times 10^{-5}\ \text{kg} \cdot \text{m}/(\text{rad/s})$.

B. Model Validation

In order to show the accuracy of the parameter estimation, a SIMULINK [16] model for the state–space description (13) was built with the parameters obtained in the experiments. Afterwards, the same signal applied to the real plant (Fig. 4) was applied to the SIMULINK model. The results are shown in Fig. 5(a) and (b) (dashed–dotted line), which show a close agreement between the model response (dashed–dotted line) and the real response (solid line). This close agreement shows that the proposed experiments are actually effective in identifying the model parameters.

VI. Conclusion

This paper has dealt with identification of the parameters of a state–space model in an undergraduate control laboratory. Experiments are proposed, and the accuracy of the model identification has been demonstrated by the estimation of all parameters of the state–space linear model of a dc motor-generator group.

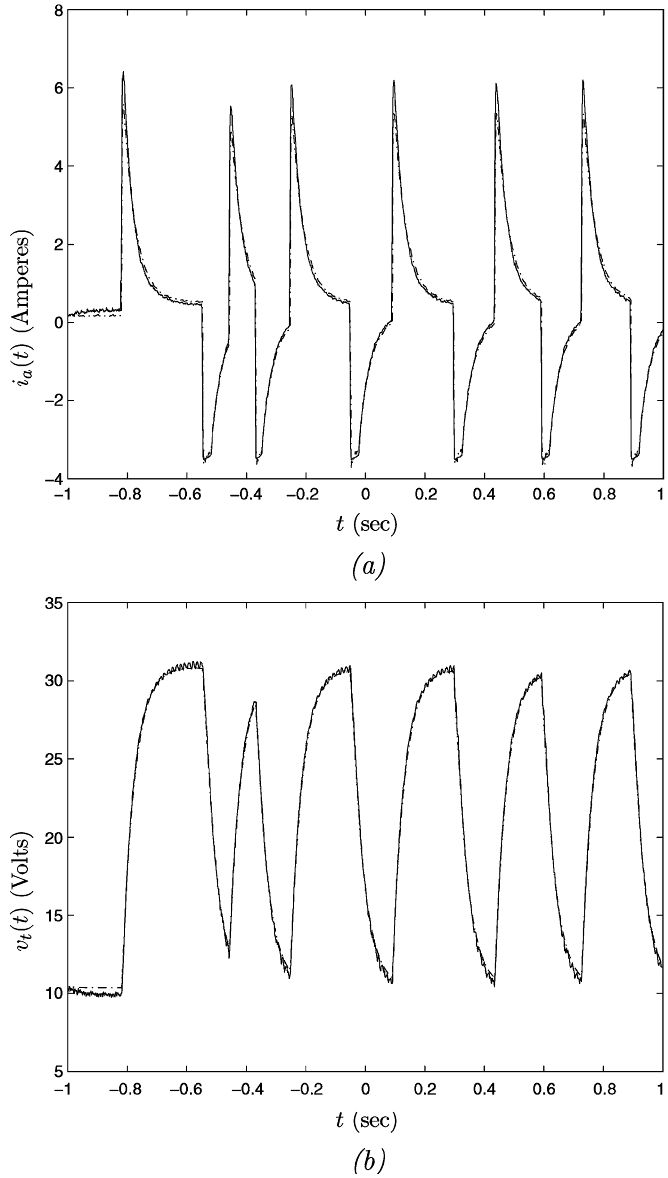

Fig. 5. Signals recorded in the experiment for the estimation of R_a, L_a, J, and f (solid lines) and the simulation results with the parameters obtained in the experiments (dashed–dotted lines).

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