Benchmark construction with application to PID controller design and implementation

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Abstract This paper addresses the issues of modelling, simulation, design, and controller implementation in an Industrial Control course. The motivation for it comes from the fact that, differently from the plants used in first courses on control systems, which usually have fast step response, plants suitable for Industrial Control courses have slow step response. The consequence of this fact is that the experiments necessary to carry out the model parameter identification take a long time to be performed. This problem can be circumvented by using a benchmark plant to resemble the behaviour of the physical system. In this paper a benchmark is constructed from a real system, having as model, a first order system with time delay, where the time delay and the time constant are both varying. Controller implementation is carried out using the PID function of a programmable logic controller (PLC).

Keywords benchmarks; control education; laboratory education; PID controllers; programmable logic controllers

The main issues dealt with in an Industrial Control course are the tuning of PID controllers for systems with large time delay based on the modelling of the plant by first or second order systems with time delay,[1] and the use of programmable logic controllers (PLC)[2] in the implementation of PID controllers. However, differently from the plants used in first courses in control systems,[3,4] which usually have a fast step response, the plants suitable for Industrial Control courses usually have a slow step response (for example, electric furnace and distillation columns). As a consequence, the experiments necessary to carry out a model parameter identification take a long time to be performed, which is inconvenient for the students who have several other tasks to complete. This problem can be circumvented by using a benchmark plant that reproduces the behaviour of the physical system with a great deal of fidelity, with the advantage that it takes only a few seconds to carry out the simulations necessary to generate the data for identification and controller design.

Benchmark problems in the context of control theory have attracted a lot of attention.[5–9] In the IFAC 1993 World Congress, researchers were challenged to design control systems for a benchmark plant, where the only available information was that the plant was a noisy time-varying single-input single-output (SISO) system working at three different operating points (additional information such as saturation levels and Bode plots were available upon authors' request). Several solutions to the IFAC 93 benchmark were proposed and published in *Automatica*.[10–15] In the context of control education, to the authors' knowledge, benchmarks have only been considered in Refs 16 and 17; the latter only considers the benchmark problem as part of a chemical process control course without considering a specific benchmark plant.

The purpose of this paper is to construct a benchmark from a real system (a resistive electric furnace). This system is modelled here as a first order system with time delay, where both the time delay as well as the time constant are varying, being dependent on the voltage applied to the furnace. The step responses obtained from both the model and the real system are compared, showing the effectiveness of the model. In addition, since PI and PID design and practical implementation are also important issues in an Industrial Control course, the paper also presents the details involved in the design of PI and PID controllers for the benchmark plant and its implementation using the PID function of a programmable logic controller (PLC); the results obtained from simulation and those from the real system are also compared.

This paper is structured as follows. The next section provides a general description of the feedback system considered in the paper. A mathematical model for the electric furnace is then obtained, and all experiments necessary to construct a benchmark plant which resembles the physical system are presented. The tuning of PI and PID controller parameters, the simulations using the benchmark plant and the actual controller implementation using a PLC are given. Conclusions are drawn in the final section.

A temperature control system for an electric furnace

Bearing in mind that, in the context of an Industrial Control course, a benchmark only makes sense if it is meant to replace the physical system in simulations to help the students in the design stage and to evaluate the compensated system performance in advance; consider a temperature control system for an electric furnace whose schematic diagram is shown in Fig. 1. The temperature measurement inside the furnace is made using a thermocouple that produces an output voltage value proportional to the temperature. Since the voltage generated by the thermocouple is low, when compared to the input voltage range admitted by the PLC, it is necessary to

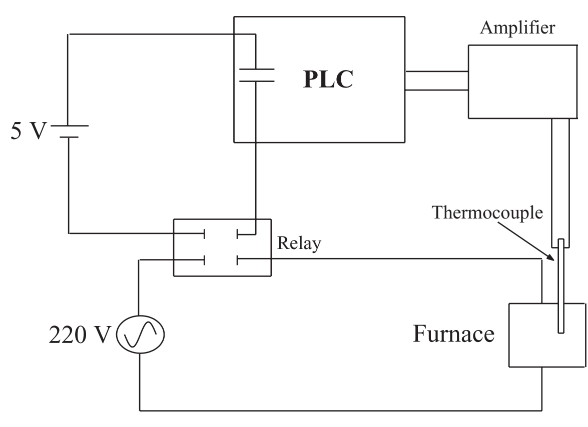


Fig. 1 *A temperature control system of a resistive electric furnace.*

use an amplifying circuit to increase this voltage. The amplified signal is then acquired by the PLC and compared with the reference value. Then, using the PID function of the PLC, a value for an internal control variable (the output of the PID function) is obtained. This value is then processed by the PLC, generating an 'on/off' type signal that drives the control circuit of a solid-state relay, and causes the furnace to be fed (or not) by a sinusoidal voltage source of 220 V (r.m.s.). The main characteristics of the process elements are described in the sequel.

Resistive electric furnace

The main characteristics of the resistive electric furnace described in this paper are as follows: maximum temperature equal to 1150°C, rated voltage equal to 220 V, rated current equal to 1.6 A, and rated frequency equal to 60 Hz.

Thermocouple

Due to the maximum temperature of the furnace (1150°C), the temperature sensor used is a K-type thermocouple, allowing temperature measurements in the range from 0 to 1100°C. The mathematical model for this element is given by

$$\upsilon(t) = K_t \theta(t) + b \tag{1}$$

where $\upsilon(t)$ is the output voltage of the thermocouple, K_t is the gain, $\theta(t)$ is the temperature inside the furnace and b is a constant. For the thermocouple used in the course described in this paper, it follows that $K_t = 41.2$ mV/°C and $b = -0.985$ mV.

Programmable logic controller (PLC)

The PLC used in the implementation of the PID function is the PLC SLC5/02 manufactured by Allen-Bradley, which is programmed using the APS programming language[18] and comprises an analog input module that allows the reading of voltages in the range 0 to 5 V, and a digital output module. The PID controller parameters to be tuned are the proportional, integral and derivative gains and can vary only within the ranges given in Table 1; thus imposing restrictions to PID controller implementation, that is, not all PI or PID controllers whose parameters are obtained from some tuning technique can be implemented exactly. It is also important to note that the PID function of the PLC executes calculations with integer binary values, transforming the input voltage into values in the range 0–16 383, and the PID function output value (control variable) is also in the same range.

After the PID function calculates the value of the control variable, the PLC reads

TABLE 1 *Proportional, integral and derivative gains for Allen-Bradley SLC5/02 PLC*

	Range	Minimum step
Proportional gain	0–25.5	0.1
Integral gain	0–1530 (s)	6 (s)
Derivative gain	0–153 (s)	0.6 (s)

this value and transforms it into values of contact opening time. Therefore, the PLC digital output is a pulsating signal of variable pulse width. This signal is used to drive a solid-state relay.

Amplifier

From eqn (1), it can be seen that for the allowed temperature values for the furnace (up to 1100°C), the thermocouple output voltage varies from approximately 0 to 0.044 V. Since the PLC analog inputs accept values between 0 and 5 V, it is advisable, for better precision of the control system, to amplify the thermocouple output signal about 100 times. This amplification is obtained with two cascade connected operational amplifiers of type 741 or LF356, both being in the inverting configuration with gain equal to −10.

Solid-state relay

This power circuit element has four terminals: two of them are hardwired to the PLC and the other two are hardwired to a 220 V voltage source and to the source circuit of the furnace. Its operation is simple: when the PLC output is such that the contact closes, the drive circuit is fed by a voltage of 5 V, as shown in Fig. 1, which causes the furnace to be fed by a 220 V sinusoidal voltage. Conversely, when the PLC output orders the contact to open, the voltage on the drive circuit terminals of the solid-state relay is 0 V; as a consequence, no voltage is supplied to the furnace.

Replacing the electric furnace with a benchmark

Theoretical background

A stable first-order system with time delay can be modelled as

$$G(s) = \frac{Ke^{-Ts}}{\tau s + 1}, \tag{2}$$

where K denotes the d.c. gain, T is the time delay (also known as dead time or transport delay) and $\tau > 0$ is the system time constant. Let $u(t)$ and $y(t)$ be the input and output signals for the system with transfer function (2). If the system is excited by a step with amplitude A, it can easily be shown that the system output is given as

$$y(t) = \begin{cases} KA\left[1 - e^{-\frac{1}{\tau}(t-\tau)}\right], t \geq T \\ 0, t < T \end{cases}, \tag{3}$$

whose plot is shown in Fig. 2.

The parameters K, τ and T of the model given in eqn (2) can be estimated with the help of eqn (3) and Figs 2 and 3. To estimate the d.c. gain K, notice from eqn (3) that $y_\infty = \lim_{t\to\infty} y(t) = KA$ and, therefore, the d.c. gain can be written as

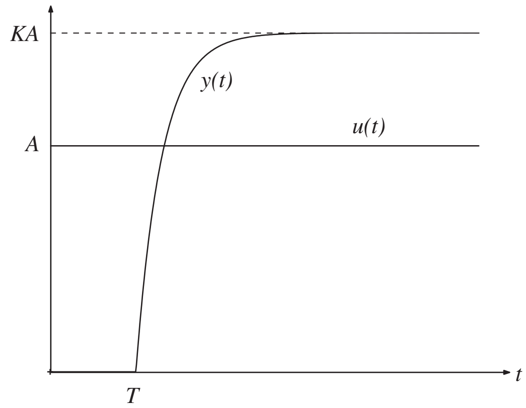


Fig. 2 *Step response of a stable first order system with time delay.*

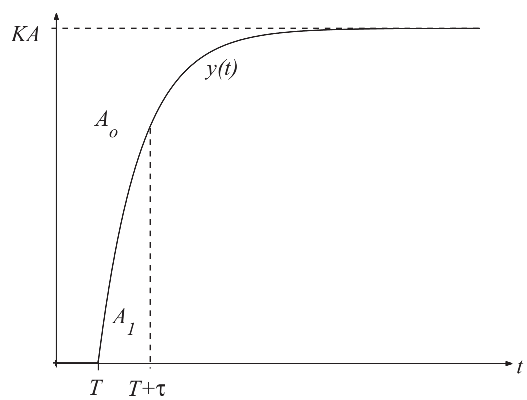


Fig. 3 *Method of areas for the estimation of* T *and* τ.

$$K = \frac{y_\infty}{A}.$$

The output signal $y(t)$ is, in general, quite noisy in practice. In this case, the value of y_∞ that minimises $\|e\|_2$, where

$$\underline{e} = [\, y_{\infty_1} - y_\infty \;\; y_{\infty_2} - y_\infty \;\cdots\; y_{\infty_n} - y_\infty \,]$$

and y_{∞_k}, $k = 1, 2, \ldots, n$ are the values of $y(t)$ for discrete-time instants $t_k > t_s$, with t_s being a time instant for which the system response can be considered as in steady-state, is given by

$$y_\infty = \frac{1}{n}\sum_{k=1}^{n} y_{\infty_k}.$$

An automatic way of obtaining the values of T and τ can be developed with the help of Fig. 3.[1] According to this method, known as the method of areas, it is enough to calculate the areas A_0 and A_1. The area A_0 can be calculated directly from eqn (3), being given by

$$A_0 = \int_0^{\infty} [KA - y(t)]\,\mathrm{d}t = KA(T + \tau). \tag{4}$$

Therefore, since $y_\infty = KA$, then $T + \tau$ can be determined; thus defining the integration upper limit necessary to calculate A_1. Straightforward calculations lead to

$$\tau = \frac{A_1 e}{KA}. \tag{5}$$

Finally, substituting the right-hand side of eqn (5) into eqn (4), it results in

$$T = \frac{A_0}{KA} - \tau = \frac{1}{KA}(A_0 - A_1 e).$$

Hence, to obtain a first-order model with time delay for a system whose step response has the shape shown in Fig. 2, the following algorithm can be utilised. It is worth remarking that the proposed algorithm can be easily implemented using Matlab.[19]

Algorithm 1
Step 1 Apply a step of amplitude A to the system and record the response $y(t_k)$, $k = 0, 1, \ldots, n$, where n is the number of sampled time instants.

Step 2 Choose a time instant t_s such that for all $t_k \geq t_s$ the output remains constant, apart from small high frequency oscillation around this constant value, and for all times $t_k \geq t_s$ define $y(t_k) = y_{\infty_k}$. Find

$$y_\infty = \frac{1}{n}\sum_{i=1}^{n} y_{\infty_i}$$

and compute the d.c. gain

$$K = \frac{y_\infty}{A}.$$

Step 3 Compute the area A_0, numerically, taking t_s as the integration upper limit and determine

$$T + \tau = \frac{\mathrm{A}_0}{y_\infty}.$$

Step 4 With the value of $T + \tau$ calculated in the previous step, compute the area A_1, numerically, and obtain

$$\tau = \frac{A_1 e}{KA}.$$

Step 6 Compute the time delay

$$T = \frac{1}{KA}(A_0 - A_1 e).$$

Step 7 Repeat steps 1 to 6 for step inputs of different amplitudes.

Step 7 of algorithm 1 leads to the computation of different values of the d.c. gain, the time delay and the time constant, making possible the identification of existing non-linearities in the system.

The benchmark construction

In the development of a benchmark, the first step is to choose a mathematical model for the physical system. Since electric furnaces generally have a step response with the shape shown in Fig. 2, a first order model with time delay, such as that given in eqn (2), is adopted.

According to step 7 of algorithm 1, the identification of the parameters K, T and τ is carried out by exciting the furnace with sinusoidal voltages having r.m.s. values equal to 20, 40, 60, 80, 100, 120, 140, 160, 180, 200 and 220 V (60 Hz). It is important to emphasise that, since the furnace is purely resistive, excitation with sinusoidal voltages is equivalent to applying steps of voltages with amplitudes equal to the r.m.s. values of the sinusoidal voltages applied to the furnace. The responses to the sinusoidal voltages of 40, 80 and 140 V are depicted in Fig. 4. With the view to highlighting the existence of a time delay in the responses, the initial time instants of the response are again shown in Fig. 5. It can be verified from the plots that the furnace has indeed the characteristics of a stable first-order system with time delay. Furthermore, it is easy to see how slow the furnace step response really is, which justifies, as far as time spent by students in lab is concerned, the need for a benchmark for this system.

Proceeding according to algorithm 1, the values of K, T and τ can be obtained from the responses to the voltages applied. The results are listed in Tables 2, 3 and 4, from which it is possible to see that all of these parameters depend on the value of the voltage applied to the furnace. As a consequence, the model of eqn (2) cannot be used directly for the benchmark, since it assumes that the values of K, T and τ are fixed and do not vary for different step amplitudes applied to the system. To circumvent this problem, these parameters are expressed in terms of polynomials of the voltage applied to the furnace, that is, $K = K(V)$, $T = T(V)$ and $\tau = \tau(V)$. The coefficients of these polynomials are obtained using least-squares to fit the points (V_i, K_i), (V_i, T_i) and (V_i, τ_i), for V_i taking the values 20, 40, 60, 80, 100, 120, 140, 180, 200 and 220, to polynomials of appropriate degrees.

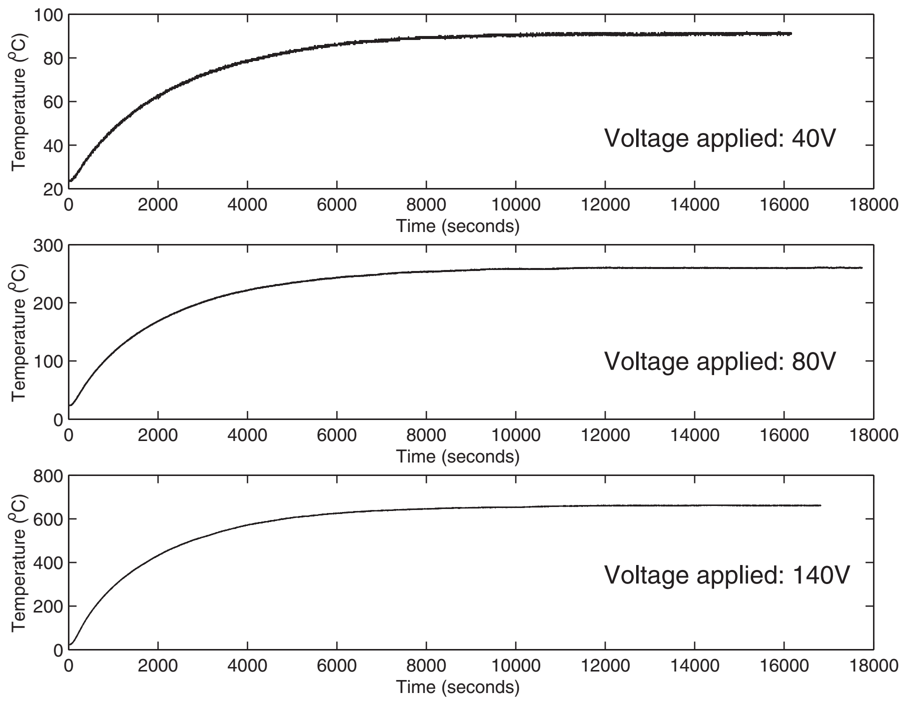


Fig. 4 *Furnace responses to sinusoidal voltages.*

TABLE 2 *Experimental results for the estimation of gain K*

Applied voltage (V)	Steady-state temperature (°C)	Temperature change (°C)	K
20	40.77	16.86	0.8432
40	90.92	67.01	1.6753
60	167.12	144.41	2.4068
80	259.99	236.66	2.9583
100	374.95	351.62	3.5162
120	503.00	479.09	3.9924
140	660.63	637.30	4.5522
180	920.28	895.79	4.9766
200	1021.34	998.62	4.9931
220	1100.87	1078.73	4.9033

Least-squares fitting for $K = K(V)$, $T = T(V)$ and $\tau = \tau(V)$

Let (x_i, y_i), $i = 1, 2, \ldots, n$ denote pairs of points in $\mathbb{R}^2$ and assume that a polynomial of degree q is to be fitted to these pairs in a least-squares sense. This problem is widely known in the literature and consists of determining the coefficients a_i, $i = 0, 1, \ldots, q$ of the polynomial

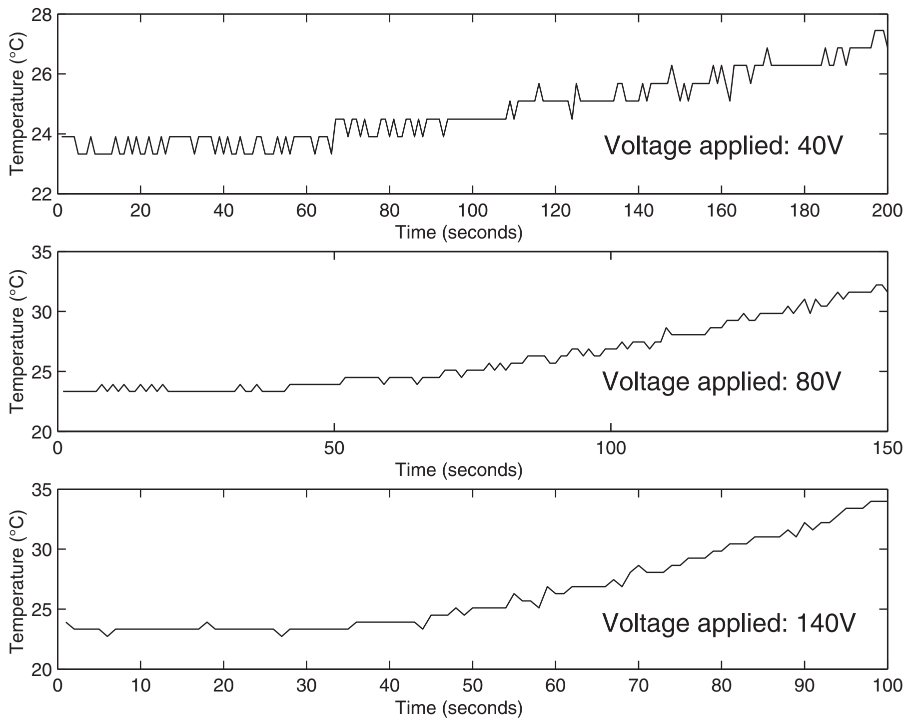


Fig. 5 *Time delay of the furnace step responses.*

TABLE 3 *Experimental results for the estimation of the time delay T*

Applied voltage (V)	*T* (s)
20	171
40	63
60	51
80	37
100	36
120	36
140	36
180	29
200	12
220	10

TABLE 4 *Experimental results for the estimation of the time constant* τ

Applied voltage (V)	τ (s)
20	1857.64
40	2241.66
60	2282.31
80	2200.12
100	2148.74
120	1957.26
140	1974.11
180	1816.79
200	1531.22
220	1394.77

$$y = a_0 x^q + a_1 x^{q-1} + \ldots + a_{q-1} x + a_q$$

such that the Euclidean norm of the error $\underline{\mathbf{e}} = \underline{\mathbf{Ax}} - \underline{\mathbf{y}}$ is minimum, where

$$\mathbf{A} = \begin{bmatrix} x_1^q & x_1^{q-1} & \cdots & x_1 & 1 \\ x_2^q & x_2^{q-1} & \cdots & x_2 & 1 \\ \vdots & \vdots & & \vdots & \vdots \\ x_n^q & x_n^{q-1} & \cdots & x_n & 1 \end{bmatrix}, \underline{\mathbf{x}} = \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_q \end{bmatrix} \text{ and } \underline{\mathbf{y}} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}.$$

Since $q >> n$, it happens that, in general, the rank of $\mathbf{A}$ is equal to $q + 1$, and the solution that minimises $\|e\|_2$ is given by

$$\underline{\mathbf{x}} = (A^t A)^{-1} A^t \underline{y}. \tag{6}$$

Direct application of least-squares fitting to the Cartesian pairs obtained from the respective values given in Tables 2, 3 and 4, leads to

$$\begin{aligned} K(V) &= -1{,}8307\times10^{-7}V^3 - 5{,}3245\times10^{-5}V^2 + 0{,}0429V, \\ T(V) &= 1{,}0704\times10^{-6}V^4 - 6{,}1090\times10^{-4}V^3 + 0{,}1215V^2 - 9{,}9727V + 320{,}5462, \\ \tau(V) &= 2{,}8325\times10^{-4}V^3 - 0{,}1414V^2 + 16{,}2702V + 1.680{,}9321. \end{aligned} \tag{7}$$

Notice that the polynomial $K(V)$ does not have the constant term, which is due to the fact that there is no change in the temperature inside the furnace when a voltage of 0 V is applied to it. It is worth remarking that, in this case, the last column of matrix $\mathbf{A}$ must be removed prior to solving eqn (6) to guarantee that the constant term of the polynomial fitted using least-squares be identically zero, that is, $a_q = 0$. Furthermore, notice that vector $\underline{\mathbf{x}}$ has, in this case, q elements, but it must be remembered that the last coefficient is zero when forming the polynomial $y(x)$. The effectiveness of the application of the least-squares fitting to the data given in Tables 2, 3 and 4 can be verified in Fig. 6, where it can be seen the plots obtained with the polynomials $K(V)$, $T(V)$ and $\tau(V)$, given by eqn (7) (solid lines) and also the points

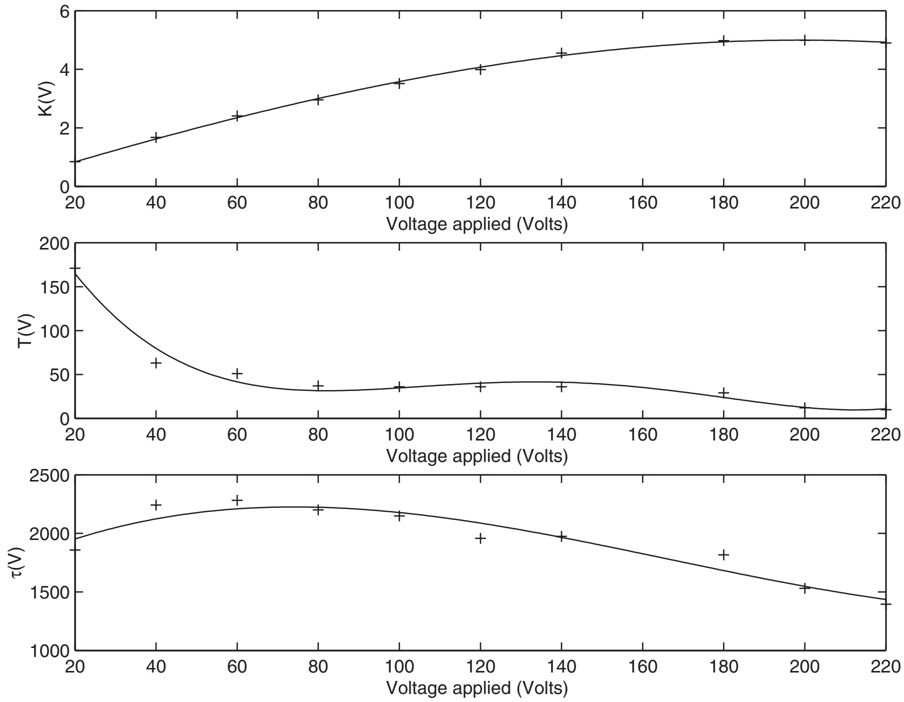


Fig. 6 *Comparison between experimental data and from the polynomials.*

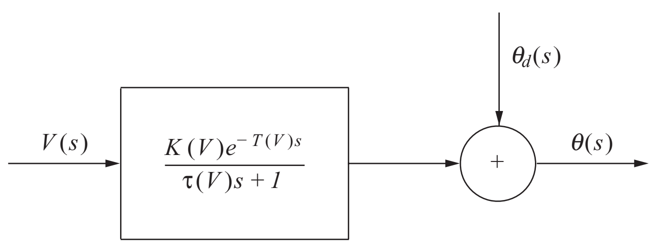


Fig. 7 *Block diagram for the benchmark.*

(V_i, K_i), (V_i, T_i) and (V_i, τ_i) formed with the first and last columns of Table 2, and directly from Tables 3 and 4, respectively.

The results above lead directly to the benchmark shown in Fig. 7, where $V(s)$, $\theta_d(s)$ and $\theta(s)$ are, respectively, the Laplace transforms of the input voltage, the environment temperature and the temperature inside the furnace and K, T and τ are expressed according to eqn (7). It is worth remarking that, as a benchmark, this model is not to be used analytically, but for simulation. In this regard, Simulink is an adequate tool to implement the block diagram of Fig. 7. Another point to be stressed is that, as remarked previously, although the voltage applied to the furnace is sinusoidal, the input signal in the simulation should be a step with amplitude equal to the r.m.s. value of the sinusoidal voltage applied in practice. If one is interested in applying sinusoidal signals to the benchmark, another block should be added to the input to convert the sinusoidal signal into a d.c. signal. Finally, notice that, as the environment temperature can be considered to be constant during the experiment, $\theta_d(s) = \theta_a/s$, where θ_a is the value of the environment temperature in degrees Celsius.

Model validation

To show that the benchmark developed in the previous subsection closely resembles the behaviour of the electric furnace, the results obtained from experiments carried out with the real furnace are compared with those obtained from simulations with the benchmark. The results are plotted in Figs 8(a) and 8(b), from which it is possible to make comparisons between the responses of the real system (solid lines) and those obtained from the benchmark (dashed lines). It can be seen that the steady-state values and the delays in each case are fairly close to those observed in the real system.

Temperature control of the furnace

Tuning PI and PID for critically damped step response

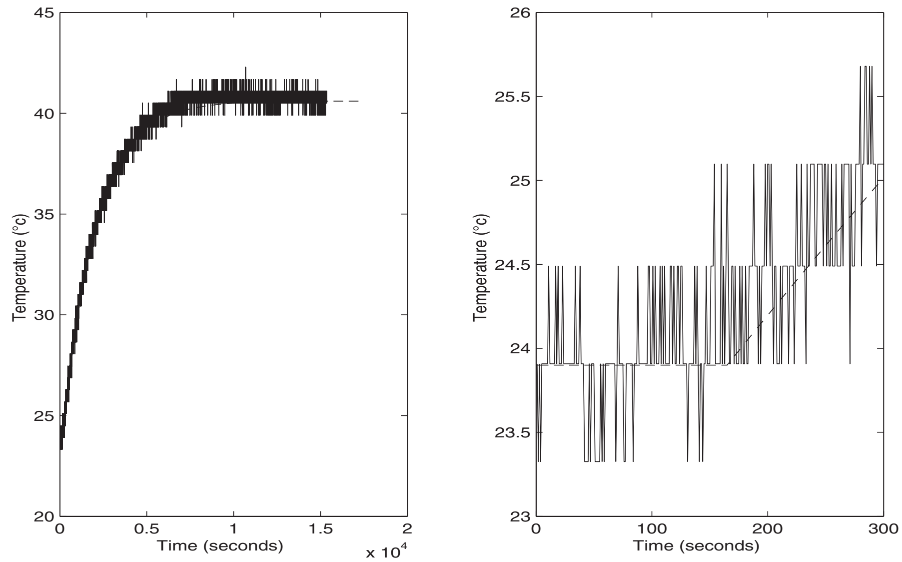
Controller implementation is also an important issue in an Industrial Control course. Thus to approach this topic, this paper describes the practical implementation of the temperature control system using the PID function of the PLC SLC5/02 (Allen-Bradley). The performance specifications are as follows: (i) percent overshoot approximately equal to zero; and (ii) settling time of the system response in closed-loop smaller than that of the open-loop system. Requirement (i) is due to the fact that overheating must be avoided in controlling furnace temperature.

It is well known that the output signal of a PID controller is a linear combination of the proportional, integral and derivative components, that is

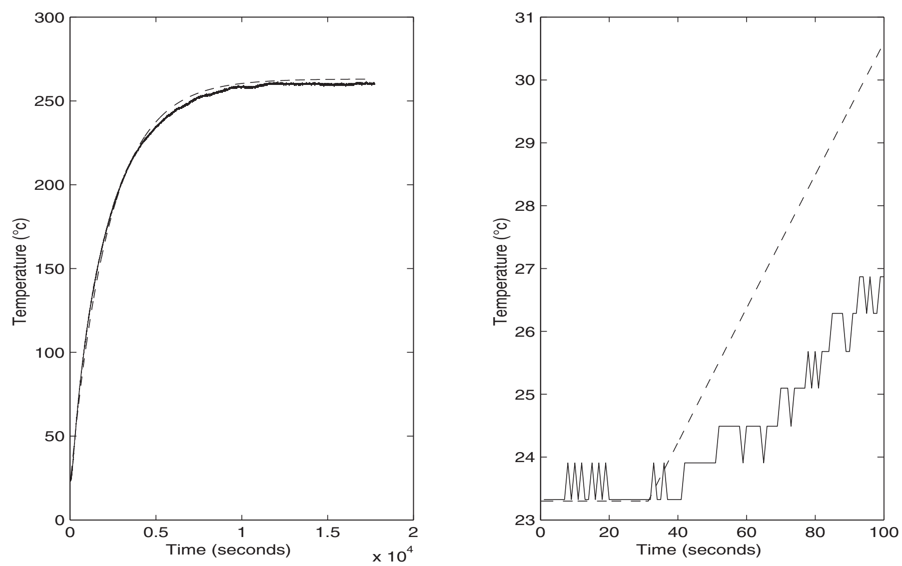
$$u(t) = K_p\left[e(t) + \frac{1}{T_i}\int_0^t e(\lambda)\,\mathrm{d}\lambda + T_d\frac{\mathrm{d}}{\mathrm{d}t}e(t)\right], \tag{8}$$

where $u(t)$ denotes the output of the PID controller, K_p represents the proportional gain, T_i is the integration time constant and T_d is the derivative time. It is important to point out that eqn (8) is already in the form appropriate for the implementation in the PLC SLC5/02.

Several techniques for tuning PID controller parameters are available in the literature, although only a few are based on the step response. One of the most widely used techniques is the well known Ziegler-Nichols Method (ZN method),[20] which is based on the step response and does not require a model for the plant. It is known that the step responses of systems compensated with PI and PID controllers tuned according to the ZN method are highly oscillatory and with high percentage overshoot. Another important PID parameter tuning technique, for systems with time



(a)



(b)

Fig. 8 *Experimental and simulation results.*

delay (which is the case of the system considered in this paper) is the Zhuang-Atherton method (ZA method).[21] Nevertheless, since no restriction is posed on the response overshoot, the response obtained from systems compensated with PID controllers tuned according to the ZA method may exhibit overshoot; although not as high as those obtained when ZN controllers are deployed. With the view to circumventing these problems, a recently developed tuning technique (BM method)[22] considers the parameter settings of PI and PID controllers with the objective of limiting the response overshoot. The BM method is likewise based on the step response whose step amplitude is defined as a function of the set point. The tuning of PI and PID controller parameters, according to the BM method, is carried out as follows.[22]

Algorithm 2

Step 1 Apply a step of amplitude A to the plant and record the output $y(t)$;

Step 2 Compute the steady-state value of $y(t)$, y_∞, and the area A_0 of Fig. 3;

Step 3 To tune a PI controller, set the integral time as $T_i = A_0/(2y_\infty)$ and the proportional gain K_p as follows: (i) $K_p = A/(4y_\infty)$ for a critically damped step response; and (ii) $K_p = A[1 + (\pi/\ln\delta)^2]/(4y_\infty)$ for a percentage overshoot equal to $\delta \times 100\%$;

Step 4 To tune a PID controller, set $K_p = 0.6699A/y_\infty$, $T_i = 5A_0/(6y_\infty)$, and $T_d = A_0/(5y_\infty)$.

Step 5 With the controller embedded in the real system, increase or decrease K_p to change the transient response of the compensated system to either increase the speed of the response or to reduce the response overshoot.

Calculation of voltage V that results in the steady-state furnace temperature equal to the desired set point

According to step 1 of algorithm 2, the reaction curve (step response) must be initially obtained, which requires a knowledge of the voltage value to be applied to the furnace such that the output steady-state value be approximately equal to 600°C. Assuming that the environment temperature is equal to $\theta_d = 23.2$°C, then the benchmark gain $K(V)$ must be equal to

$$K(V) = \frac{\Delta\theta}{V} = \frac{600 - 23.2}{V} = \frac{576.8}{V}.$$

Comparing this equation and equ (7a), yields

$$-1.8307\times10^{-7}V^4 - 5.3245\times10^{-5}V^3 + 0.0429V^2 - 576.8 = 0.$$

Notice that equation above has four roots. According to Table 2, the desired solution is real and must belong to the interval [120,140]. It can be easily verified that for $V = 133.0715\,\mathrm{V}$, the steady-state temperature is $\theta_\infty = 600$°C. Therefore, a voltage step with amplitude equal to $A = 133\,\mathrm{V}$ must be applied to the furnace.

Tuning of PI and PID controllers using the benchmark

In this section, the tuning of PI and PID controllers for the temperature control is considered. To do so, assume that the furnace is required to operate in steady state at 600°C. Given the specification of percentage overshoot approximately equal to zero, the BM method is more appropriate for the tuning of the controller parameters.

There is no need to actually carry out this experiment in the real system, since the reaction curve can be obtained directly from the benchmark. The response of an input voltage equal to 133 V is presented in Fig. 9 (dotted line), from which it is possible to compute the area A_0 numerically (equal to 1 182 209.76) and to determine, using numerical interpolation, the response settling time specification (equal to 7897.8 s). In addition, since $\theta_d = 23.2$°C, it follows that $y_\infty = 576.8$°C. Therefore, according to steps 3 and 4 of algorithm 2, the PI and PID controller parameters can now be calculated, and are listed in the first two rows of Table 5. The responses obtained by compensating the benchmark plant with these controllers are shown in Fig. 9 (solid lines). Fig. 9 shows that the response of the system compensated with the PID controller is faster than that obtained with the PI controller. In fact, according to Table 6 (theoretical parameters), it can be seen that, in both cases, the percentage overshoot of the response is approximately 1%, which satisfies the approximately zero overshoot specification; however the settling time specification can only be met by the system controlled with the PID controller; approximately 1066 seconds smaller than the open-loop system settling time. It is worth noting that, according to algorithm 2, even further reduction in the closed-loop system percentage overshoot can be obtained by reducing the proportional gain K_p; however, this fine tuning must be done in loop. The consequence of reducing the gain K_p is an increase in the settling time.

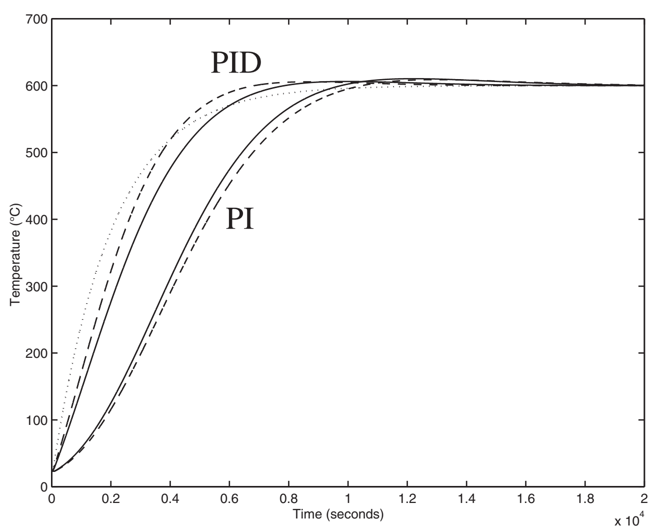


Fig. 9 *Reaction curve and simulated step responses of the compensated system.*

TABLE 5 *PI and PID parameters*

	Controller	K_p	T_i	T_d
Theoretical parameters	PI	0.0577	1024.8503	–
	PID	0.1546	1708.0839	409.9401
Parameters used in simulation (equivalent to those used in PLC)	PI	0.0543	1026	–
	PID	0.1629	1530	153
Parameters used in the PLC	PI	0.3	1026	–
	PID	0.9	1530	153

TABLE 6 *Performance indices of the feedback system*

	Controller	y_∞	t_s	P.O.
With theoretical parameters	PI	600°C	8947.3 s	1.22%
	PID	600°C	6831.28 s	1.04%
With parameters equivalent to those used in PLC	PI	600°C	9588.04 s	1.33%
	PID	600°C	5.893.24 s	0.94%

Simulation results

The restrictions imposed by the PLC on the PID controller parameters (Table 1) imply that the values of K_p, T_i and T_d obtained in the previous sub-section cannot be used directly in the implementation. Parameter T_i varies slightly due to the minimum step of the PLC integral gain, which is equal to 6 seconds, while the value of parameter T_d must be replaced with the largest allowed one (153 seconds), since the calculated value exceeds this limit. The proportional gain needs to be converted to an equivalent value since the PID function of the PLC performs calculations with integer binary values and because the output of the PID function is also a binary value.

Notice that the analogue input to the PLC must be a value between 0 and 5 V, and that the PLC input is the amplifier output, which corresponds to the output voltage of the thermocouple multiplied by 100. In addition, since the PID function of the PLC performs calculations with integer binary values (0 to 16383), then the internal value of the error in the PLC is given by

$$E_{PLC} = \frac{100\times 16382}{5}(V_{ref} - V_t) = \frac{100\times 16382}{5}(K_t\theta_{ref} + b - K_t\theta - b),$$

where V_{ref} and V_t denote, respectively, the reference and output voltage of the thermocouple, K_t and b are, respectively, the angular and linear coefficients of the thermocouple model and θ_{ref} and θ correspond, respectively, to the reference temperature (set point) and to the temperature inside the furnace. Consequently

$$E_{PLC} = \frac{100\times 16382}{5}K_t(\theta_{ref} - \theta). \tag{9}$$

In addition, notice that the proportional component in the PLC control variable is given as

$$U_{PLC} = K_p^{PLC} E_{PLC},$$

and therefore, using the right-hand side of eqn (9) in equation above, yields

$$U_{PLC} = \frac{100 \times 16382}{5} K_t K_p^{PLC} (\theta_{ref} - \theta). \tag{10}$$

The control variable (output of the PID function) is also a binary value (0 to 16383) and therefore the RMS value of the voltage that corresponds to U_{PLC} is

$$V_{ef} = \frac{220}{16383} U_{PLC}. \tag{11}$$

Thus, using eqns (10) and (11), the r.m.s. value of the control variable, in terms of the gain to be used in the PLC, can be written as

$$V_{ef} = \frac{220 \times 100}{5} K_t K_p^{PLC} (\theta_{ref} - \theta). \tag{12}$$

On the other hand, since the proportional component of an ideal PID is

$$V_{ef} = K_p (\theta_{ref} - \theta), \tag{13}$$

it turns out that, comparing eqns (12) and (13) and taking into account that $K_t = 41.2 \times 10^{-6}$, the value of the proportional gain that must be used in the PID function of the PLC is given as

$$K_p^{PLC} = \frac{K_p}{0.181}. \tag{14}$$

Hence, according to eqn (14), the proportional gains to be used in the implementation of the PI and PID controllers, which are equivalent to the proportional gains listed in the first two rows of Table 5 are, respectively, 0.3188 and 0.8541. However, since the parameters of the PID to be used in the PLC are restricted to the ranges given in Table 1, the values given in the last two rows of Table 5 (parameters used in the PLC) must be used in the implementation of the PI and PID controllers whose parameters are given in Table 5 (theoretical parameters).

The simulation results for PI and PID controllers with parameters equivalent to those used in the PID algorithm of the PLC (two middle rows of Table 5) are shown in Fig. 9 (dashed lines). Comparing the simulation results of the PI and PID controllers with theoretical parameters and those obtained with the parameters equivalent to those used in the PLC, it can be seen, with the help of Table 6, that the response of the PI compensated system with the PLC parameters has larger settling time than that tuned with the theoretical parameters. On the other hand, for the PID control, a substantial improvement in the performance results, and the response settling time is decreased significantly, as one can see from Table 6.

Conversion of the control variable into opening time

The use of a solid-state relay as actuator implies that additional calculations must be carried out by the PLC after the PID algorithm calculates the value of the control variable. These calculations are needed to guarantee that the r.m.s. value of the voltage applied to the real furnace is equal to the value applied to the Simulink model. In the context of an Industrial Control course, these aspects are important since the students are given an opportunity to face a realistic situation that is usually different from those encountered in the basic control laboratories. The details are given in the sequel.

Notice that the binary control variable at the output of the PID function (U_{PLC}) represents a percentage value of the maximum voltage to be applied to the furnace (220 V); its relation to the r.m.s. value of the voltage to be applied to the furnace is given in eqn (11). This value cannot be used directly since it needs to be transformed into contact opening time. Therefore, it is necessary to obtain a relationship between the contact opening time and the corresponding r.m.s. value of the voltage signal produced at the output of the solid-state relay.

Consider Fig. 10, where T_r denotes the sampling interval, that is, the time interval for updating the control variable and t_{a_i}, $i = 1, 2, \ldots$, represent the time instants when the relay contacts must open, interrupting the current supplied to the furnace. Then, the r.m.s. value of the voltage in the interval $[0, T_r]$ is given by

$$V_{ef}^2 = \frac{1}{T_r}\int_0^{t_a}\left[220\sqrt{2}\sin(377t)\right]^2 \mathrm{d}t.$$

To solve the above integral, a simplifying assumption is made, namely that t_a is equal to an entire number of cycles. This assumption is justifiable because the oscillation period of the voltage signal (1/60 s) is small when compared to the value of T_r, which,

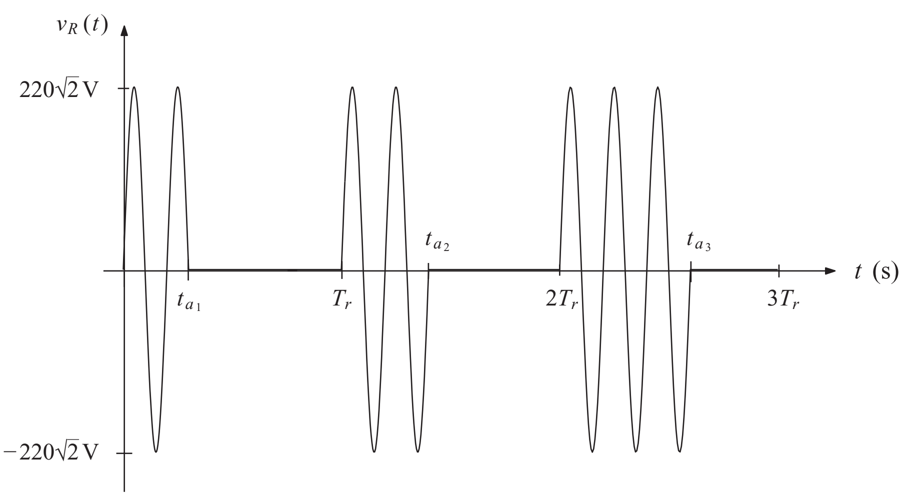


Fig. 10 *Typical curve of a solid state relay.*

since the furnace has slow step response, is of the order of seconds. It can be easily checked that, with this assumption, the integral above evaluates to

$$V_{ef}^2 = \frac{t_a}{T_r} 220^2. \tag{15}$$

Finally, substituting the right-hand side of eqn (11) into (15), yields

$$t_a = \left[\frac{U_{PLC}}{16.383} \right]^2 T_r, \tag{16}$$

which is the expression that must be programmed in the PLC to convert the value of the control variable of the PID into contact opening time. Notice that this expression is dependent on the sampling time. Since the furnace has a slow response, $T_r = 4$ s is a reasonable value.

Experimental results

The simulation results presented in the previous subsection show that the best controller for the temperature control of the real system is a PID whose parameters are given in the last row of Table 5. It is worth stressing that since the output of the PLC is an 'on/off' signal, then a new command line – eqn (16) – must be added to the PLC program to define the time instant when the contact must open. The experimental response for a reference step of 600°C is shown in Fig. 11 (spiky line). In this figure, the step response obtained from a Simulink closed-loop system having as controller and plant, respectively, a PID controller whose parameter values are given in the last row of Table 5 and the benchmark developed in this paper (smooth

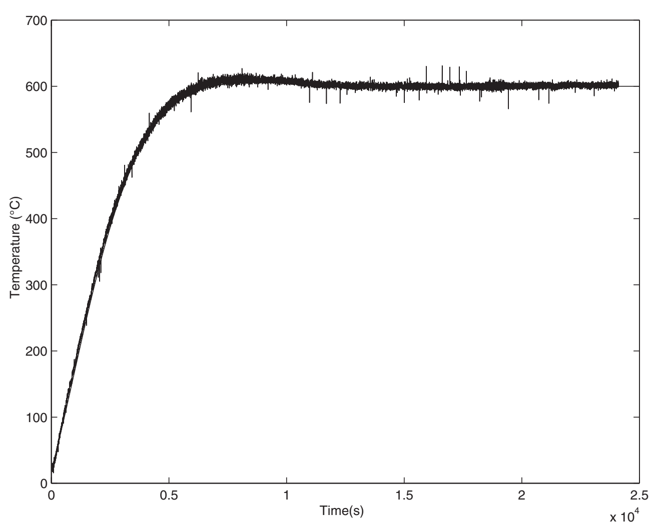


Fig. 11 *Step responses of the compensated feedback system.*

lines) is also presented. Notice that the visualisation of the curves obtained from the simulations is somewhat difficult, because of the noise introduced by the sensor. Nevertheless, it is possible to see that the experimental results and those obtained from the simulations are almost coincident, showing once again the validity of the benchmark developed here and the effectiveness of the BM as a tuning method for PID controllers when critically damped step response is a design specification.

Concluding remarks

In this paper, all the stages of the construction of a benchmark for an electric furnace were detailed. In the modelling/identification stage, experiments were proposed and a systematic way to obtain a model for the benchmark was also presented. The validity of the model was proved by comparing the simulation results obtained from the benchmark and those obtained from experiments carried out with the real furnace.

Since this benchmark was meant for an Industrial Control course, PI and PID controllers were designed, and tuned using the BM method,[22] recently proposed in the literature. Simulations were carried out with the benchmark plant using PI and PID controllers whose parameters were tuned using the BM method. As expected, the performance of the system controlled with the PID was better than that obtained using a PI controller. Furthermore, the system compensated with the PID controller was able to meet the performance specifications; therefore adopted as the controller for the real system. The compensated system performed as expected, that is, with a very low overshoot, showing the effectiveness of the BM method in the tuning of the parameters of a PID controller when critically dumped step response is a performance specification. The minute overshoot could be avoided by slightly reducing the controller gain. The controller implementation was carried out using the PID function of a PLC (Allen-Bradley SLC5/02), but could have been done using, for example, Simulink Real-Time Windows Target.

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