A laboratory for a first course in control systems

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Abstract The design of automatic control systems nowadays requires a great deal of theoretical knowledge. The consequence of this fact is that a large number of new concepts have to be introduced in a first course in control systems. However, these concepts are in general introduced somehow independently and this poses serious problems when the students are required to deal with the whole design of a control system. In this paper a control laboratory is proposed with the view to putting together all the concepts previously introduced in a theoretical course. Experiments are suggested such that all the stages of a control system design, namely modeling/identification, design/simulation and implementation are covered.

Keywords control education; laboratory education; system identification; PI controllers

In accordance with a recent paper,[1] the main objectives of control education are: (i) to provide the basis for the future control engineer to be able to deal with the design of control systems for different plants and (ii) to establish and maintain high standards in the presentation of the main concepts of control. The latter is generally the goal of a first course in control while the former, although relying on a strong background, can only be achieved with the help of a good control laboratory. It is well accepted that a good control laboratory must not only illustrate the concepts introduced in the theoretical course but have to be realistic as well.[2]

Motivated by these facts, a control laboratory[3] for the Electrical Engineering Course of the Federal University of Rio de Janeiro (UFRJ) has been developed and successfully used. The plant consists of a d.c. motor-generator group represented in Fig. 1, where $v_a(t)$ is the input voltage, $v_t(t)$ is the voltage at the tachometer terminals and $i_g(t)$ denotes the current supplied by the generator when a load is connected

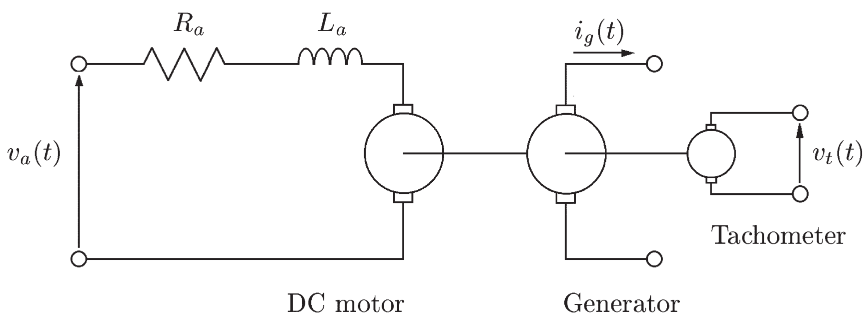

Fig. 1 *Equivalent circuit for a d.c. motor-generator group with a tachometer.*

to its terminals. The problem here is to control the shaft velocity in the presence of plant uncertainties and external disturbance signals.

A d.c. motor-generator group has been chosen for the plant and the shaft velocity as the control variable because they allow the following concepts to be illustrated:

1 Modeling;
2 Linearity, identification and measurement noise;
3 The effects of parameter identification errors and external disturbance in an open-loop control system;
4 The benefits of feedback;
5 The need for dynamic compensation.

Nowadays it is almost impossible to carry out a control system design without the help of a computational program. In this laboratory the students are strongly encouraged to use MATLAB and SIMULINK.[4] MATLAB is well known for its capacity in dealing with matrices and therefore can be used in identification, controller design and in the analysis of performance of compensated systems. SIMULINK plays an important role during controller design since it allows the system performance to be evaluated (through simulation) immediately after a controller has been designed. Furthermore, when data acquisition is carried out through a digital oscilloscope or a digital computer, SIMULINK becomes a powerful tool to compare the results obtained during simulation and those actually reached with the real plant.

This paper is structure as follows. The section below presents two linear models for a d.c. motor-generator group: a second-order model, and a simpler one, which is obtained from the previous model by neglecting the fast dynamic. The next section deals with identification, covering the following topics: (i) the definition of the static characteristics of the system and as consequence a nonlinearity (dead zone) will be introduced in the model; (ii) the definition of a region of operation in which the system can be considered linear and (iii) experiments for the determination of the gains and the system dynamics. The fourth section presents the design of a control system for the d.c. motor-generator with step input tracking and step disturbance rejection objectives. These objectives make the control problem more interesting since they can only be achieved with a dynamic compensator, more precisely, a compensator with integral action. Finally, in the fifth section, a very simple electronic network will be introduced for the physical implementation of the controller derived in the previous section. Since this laboratory is meant for a first course in control, analog devices are deployed. However, it is important to note that the same controller could be easily implemented by using a digital controller if this laboratory takes place after a course in discrete-time control systems.

Modeling

A mathematical model for the d.c. motor-generator group depicted in Fig. 1 can be obtained simply by considering the equivalent circuit of the armature-controlled d.c.

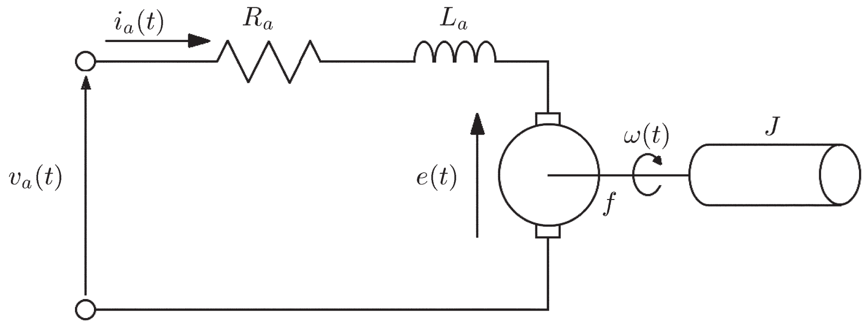

Fig. 2 *Equivalent circuit of an armature-controlled d.c. motor.*

motor of Fig. 2, where $v_a(t)$ and $i_a(t)$ denote, respectively, the input voltage and the current in the armature circuit, $\omega(t)$ is the shaft velocity and J and f are, respectively, the load inertia and the bearing friction. It is not difficult to show that:[5]

$$\Omega(s) = \frac{K_m/(R_a f)}{(\tau_e s+1)(\tau_m s+1)+K_e K_m/(R_a f)} V_a(s) - \frac{(\tau_e s+1)/f}{(\tau_e s+1)(\tau_m s+1)+K_e K_m/(R_a f)} T_d(s) \tag{1}$$

where $\tau_e = L_a/R_a$, $\tau_m = J/f$, K_m is the torque constant, K_e is the counter electro-motor force and $t_d(t)$ denotes the disturbance torque which, in this case, appears when a load is connected to the generator terminals. However, since $L_a/R_a << 1$ then for low and intermediate frequencies $\tau_e s + 1 \approx 1$ and, therefore, a simpler model for this system is given by:

$$\Omega(s) = \frac{K_a}{\tau s+1} V_a(s) - \frac{K_d}{\tau s+1} T_d(s) \tag{2}$$

where

$$K_a = K_m/(R_a f + K_e K_m),$$
$$K_d = R_a/(R_a f + K_e K_m) \quad \text{and} \quad \tau = JR_a/(R_a f + K_g K_m).$$

In order to obtain a complete model for the system, it remains to take into account the effects of the disturbance current ($i_g(t)$) and the tachometer as well. The former can be accounted for, with the help of Fig. 1, by remembering that $t_d(t) = \bar{K} i_g(t)$ while the latter is usually modeled as a constant gain system, i.e. $v_t(t) = K_t \omega(t)$. Finally, defining $K_g = \bar{K} K_d$, the transfer function which relates $v_a(t)$ and $i_g(t)$ to $v_t(t)$ can be written as follows:

$$V_t(s) = \frac{K_a K_t}{\tau s+1} V_a(s) - \frac{K_g K_t}{\tau s+1} I_g(s), \tag{3}$$

leading to the block diagram of Fig. 3.

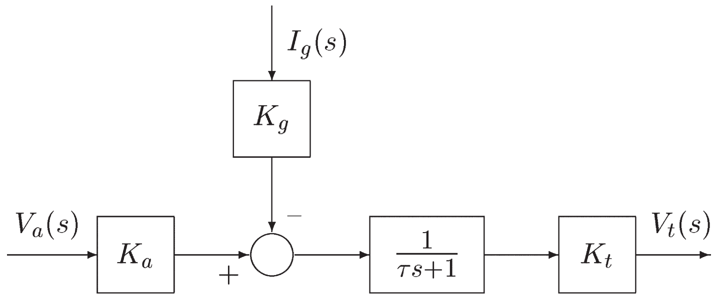

Fig. 3 *Block diagram for the d.c. motor-generator group with tachometer included.*

Identification

The model presented in Fig. 3 assumes that the plant is linear. However, in general, this is not true for all values of $v_a(t)$ and so the students are recommended to first study the static behaviour of the system and as a consequence to obtain the range of $v_a(t)$ for which the system is linear. Once a linear region of operation has been defined, the students can carry out experiments to determine the system gains and the time constant. With this in mind, experiments for model parameter identification are proposed in the sequel:

Experiment to find the linear region of operation and the static gain K_aK_t

The linear region can be determined as follows: (i) apply constant d.c. voltages (V_a) to the armature terminals and record the corresponding steady-state values of the voltages (V_t) at the tachometer terminals; (ii) plot the points (V_a,V_t) and use least-squares to fit the points to a polynomial ($p(V_a)$) of a desired order; (iii) compute the derivative of $p(V_a)$ with respect to V_a. Thus the linear region corresponds to the interval where the derivative is nearly flat. This procedure is illustrated in Fig. 4, from where it is possible to conclude that, for the d.c. motor-generator group of the control laboratory of UFRJ, the linear region is between 3 and 16 V. Note that, in addition to the determination of a linear region, this experiment has also brought to light the need for a nonlinear model in order to take into account the dead zone, shown in Fig. 4(a), which is caused by backslash and variable friction. Mathematically, the dead zone can be expressed as:

$$v_t(t) = \begin{cases} K_aK_t[v_a(t) - V_0], & v_a(t) > |V_0| \\ 0 & v_a(t) < |V_0| \end{cases} \qquad (4)$$

The consequence of the introduction of dead zone (4) is that the linear model of Fig. 3 is only valid in the linear region and, therefore, a better description of the system will be provided by a model with a block diagram such as that depicted in Fig. 5.

Finally, notice that the static gain K_aK_t and the dead zone value V_0 can also be determined from the data obtained in this experiment. The computation of K_aK_t can

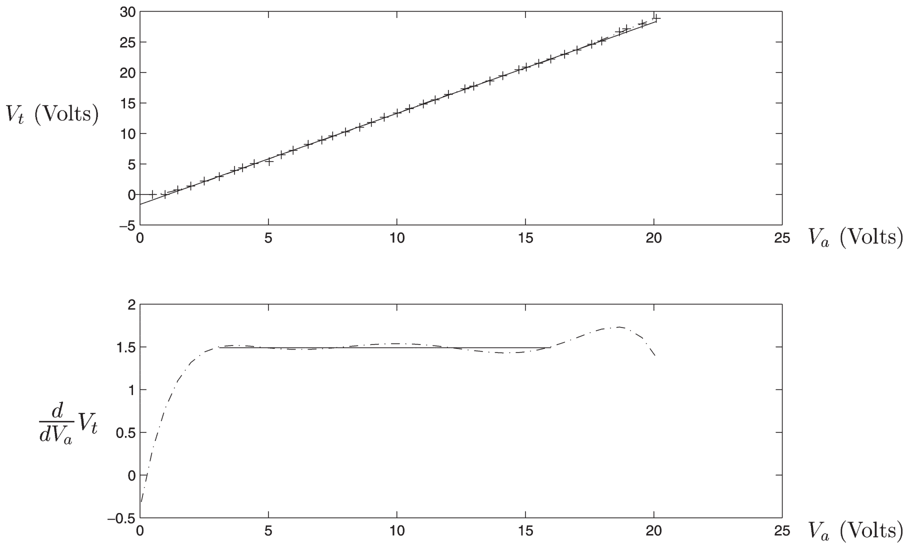

Fig. 4 *Data obtained from the experiment to determine the linear region: (a) Points* (V_a, V_t) *(+),* p (V_a) *(–.) and* $V_t = K_aK_tV_t + b$ *(–); (b)* $dp(V_a)/dV_a$*(–.) and* K_aK_t*(–).*

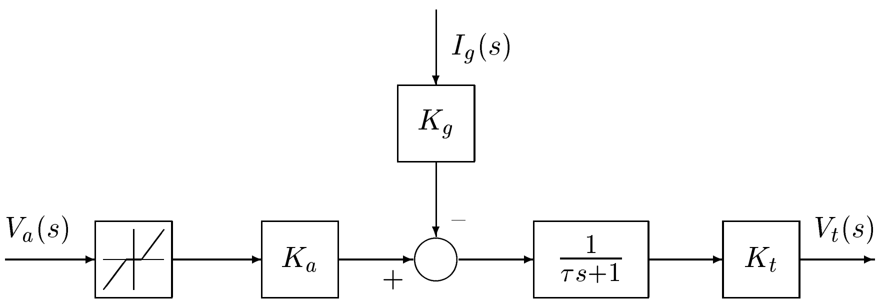

Fig. 5 *Block diagram with a nonlinear block (dead zone).*

be carried out in several ways. The simplest one is to take the mean value of $dp(V_a)/dV_a$ for the values of V_a in the linear region determined above.

Once K_aK_t has been calculated, we can use least-squares theory to obtain the following expression for V_0:

$$V_0 = \sum_{i=1}^{n} \frac{1}{n} V_{t_i} - K_a K_t V_{a_i} \tag{5}$$

where i ranges for values of (V_a, V_t) belonging to the linear region. The application of this procedure to the d.c. motor-generator group of UFRJ has led to the following values: $K_aK_t = 1.4906$ and $V_0 = 1.0857$ V.

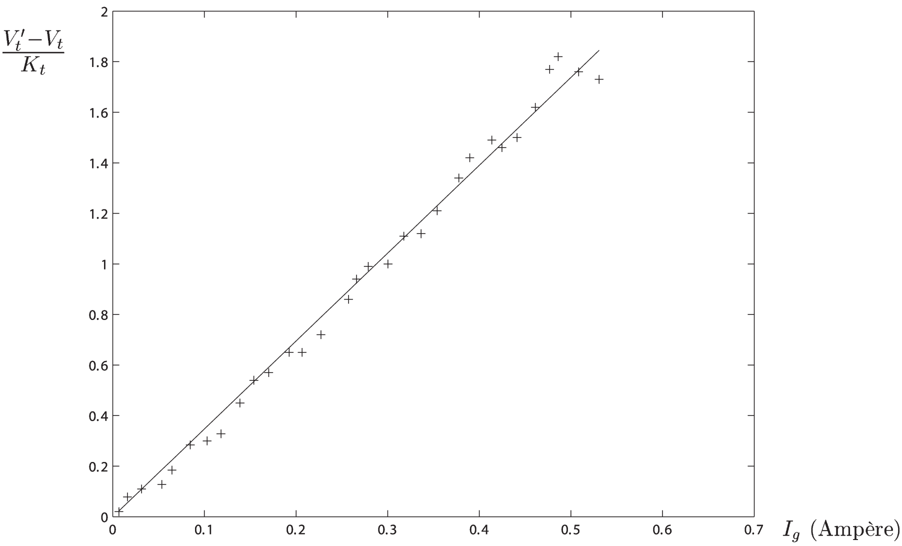

Fig. 6 *Data obtained from the experiment for the determination of* K_g.

Experiment to determine the parameters K_a and K_g

With the value of K_aK_t previously determined and since the tachometer gain K_t is in general known* then K_a can be given by:

$$K_a = K_aK_t/K_t\,. \tag{6}$$

The tachometer used in the control laboratory of UFRJ has a gain $K_t = 0.007$ V/rpm and therefore $K_a = 218.01$ rpm/V.

The gain K_g is determined with the help of (3) as follows: (i) apply a step input of amplitude V_a and note that its steady-state value is $V_t = K_aK_tV_a$; (ii) with the same input, consider a perturbation of amplitude I_g and note that the steady-state value is now $V_t' = V_t - K_gI_g$ and therefore

$$\frac{V_t' - V_t}{K_t} = K_gI_g \tag{7}$$

(iii) repeat steps (i) and (ii) for several values of V_a and I_g and use least square fitting to find K_g. The application of this procedure to the d.c. motor-generator group of UFRJ is illustrated in Fig. 6 and has led to $K_g = 496.54$ rpm/A.

Experiments for the determination of the system time constant τ

From (3), assuming that $I_g(s) = 0$, the transfer function from $V_a(s)$ to $V_t(s)$ can be written as:

*If K_t is unknown then it can be determined by least square fitting of the points (ω_i, V_{t_i}), $i = 1, 2, \ldots,$ n, where ω is the shaft velocity and V_t is the voltage at the tachometer terminals.

$$G(s) = \frac{K_a K_t}{\tau s + 1}, \tag{8}$$

which represents a first order system with time constant τ. There are several ways to determine τ but basically these methodologies depend on either step or sinusoids responses,[6] as will be shown in the sequel.

Identification of τ from the transient response of a step input

The response of the first order system (8) to a unit step input is given by:

$$v_t(t) = K_a K_t \left(1 - e^{-t/\tau}\right). \tag{9}$$

Notice that:

$$\frac{d}{dt} v_t(t)\Big|_{t=0} = \frac{K_a K_t}{\tau}, \tag{10}$$

and, therefore, a straight line with slope equal to (10) would take τ (units of time) to reach the steady-state value of (9). This allows us to propose the following algorithm for the determination of τ: (i) with the system operating in the linear region, apply a step input with amplitude such that the system remains operating in the linear region and acquire the output signal; (ii) compute V_{t_o} and $V_{t_{inf}}$, the steady-state output mean values before and after the application of the step; (iii) use least-squares to fit the very first points of the transient response to a straight line which necessarily goes over the point $(0, V_{t_o})$ and let its slope be α; (iv) the system time constant will be given by:

$$\tau = \frac{V_{t_{inf}} - V_{t_o}}{\alpha} \tag{11}$$

The application of the procedure described above to the d.c. motor-generator group of UFRJ is illustrated in Fig. 7, leading to $\tau = 0.0437\,s$.

Identification of τ from the frequency response

Since the system has been modeled as a first order (eqn (3)), the students only have to find the corner frequency of the Bode magnitude diagram which is numerically equal to $1/\tau$. The plot of Bode diagrams requires the knowledge of the system frequency response, which can experimentally be obtained by applying sinusoidal signals of different frequencies $v_a(t) = V_{a_m} + V_{a_{max}} \sin(\omega_f t)$, where V_{a_m} and $V_{a_{max}}$ are such that $[V_{a_m} - V_{a_{max}}, V_{a_m} + V_{a_{max}}]$ is in the linear region. The output corresponding to this input will be $v_t(t) = V_{t_m} + V_{t_{max}} \sin(\omega_f t + \phi)$ and, theoretically, the gain in dB can be computed as follows:

$$\left|G(j\omega)\right|_{dB} = 20 \log \frac{V_{t_{max}}}{V_{a_{max}}}. \tag{12}$$

However, as shown in Fig. 8, the presence of measurement noise makes the computation of $V_{t_{max}}$ more complicated and therefore the students should be encouraged

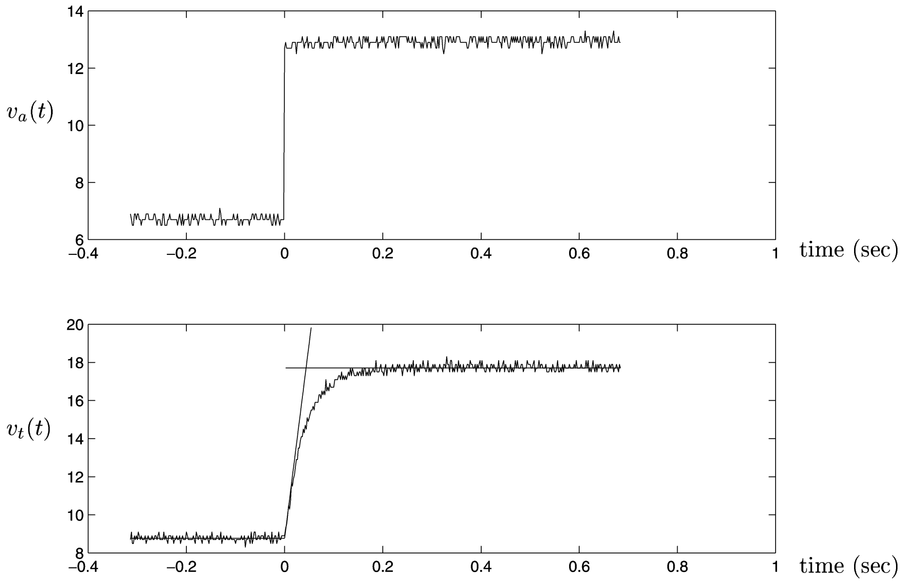

Fig. 7 *Data obtained from the experiment for the determination of* τ *from the transient response of a step input.*

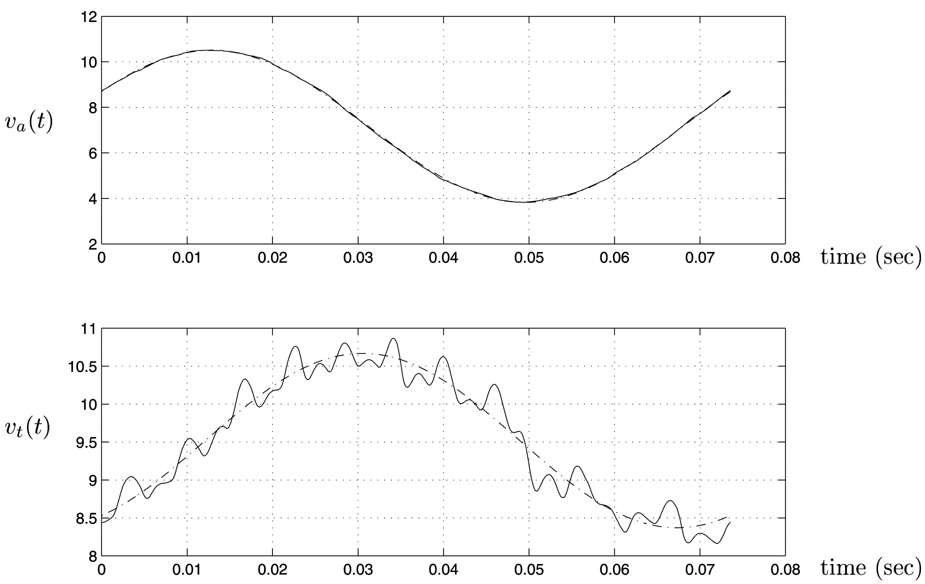

Fig. 8 *Input and output signals (solid lines) and the fundamental components (dash-dotted lines) of the corresponding Fourier series.*

to use Fourier series to find the fundamental component of the signal. Notice that a periodic function $v(t)$ with period $T = 2\pi\omega_f$ defined in an interval $[t_0, t_0 + \mathrm{T}]$ has a Fourier series representation given as:

$$v(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_f t) + b_n \sin(n\omega_f t), n = 1, 2, \ldots$$

where

$$a_0 = \frac{1}{T}\int_{t_0}^{t_0+T} v(t)\,dt, \;\; a_n = \frac{2}{T}\int_{t_0}^{t_0+T} v(t)\cos(n\omega_f t)\,dt \text{ and } b_n = \frac{2}{T}\int_{t_0}^{t_0+T} v(t)\sin(n\omega_f t)\,dt.$$

Therefore, in order to obtain the d.c. and the fundamental frequency components of the input and output signals, all is needed is to compute a_0, a_1 and b_1, which are given by the formulae above. However, there is a minor problem: the functions $v_a(t)$ and $v_t(t)$ are not known analytically but described by sets of pairs $[t_k, v_a(t_k)]$ and $[t_k, v_t(t_k)]$ for $t_k \in [t_0, t_0 + \mathrm{T}]$. With the help of MATLAB, this problem can be easily handled and the students may write their own functions to compute a_0, a_1 and b_1. This function should have the following steps: (*i*) collect one period from the acquired points (this can be easily done with the help of functions such as `ginput` and `find`; (*ii*) compute $\sin(\omega_f t_k)$ and $\cos(\omega_f t_k)$ for all t_k in the interval $[t_0, t_0 + \mathrm{T}]$; (*iii*) use the vector operation `.*` together with the MATLAB function `trapz` to find the values of a_0, a_1 and b_1 for $v_a(t)$ and $v_t(t)$. Figure 8 shows one period of the input and output signals for the dc motor-generator group of UFRJ for approximately 1.35 Hz. Note that the input signal is very close to an ideal sinusoid whereas the output has been corrupted by noise, showing the need for computing the Fourier series. The reader should note that the use of Fourier analysis also makes easier the computation of the phase difference between output and input signals. Although this is not necessary in the present work, this point should be made clear to the students. The identification of τ from the Bode diagram has also been carried out for the dc motor-generator group of UFRJ leading to the diagram depicted in Fig. 9. The value of τ obtained from this experiment was approximately 0.0416 s, being therefore very close to that estimated from the step response.

Controller design

Once a mathematical model for the d.c. motor-generator group has been obtained and all the parameter values have been determined, the next step is to design a controller which satisfies the following requirements:

1 Stability;
2 Zero Steady-state error, i.e. the voltage in the tachometer terminals $v_t(t)$ must be, in steady-state, equal to a given reference voltage $v_{t_r}(t)$ (which corresponds to the desired angular velocity);
3 Low sensitivity to identification errors in the model parameters;
4 Disturbance rejection, i.e. for a load connected to the generator terminals, the

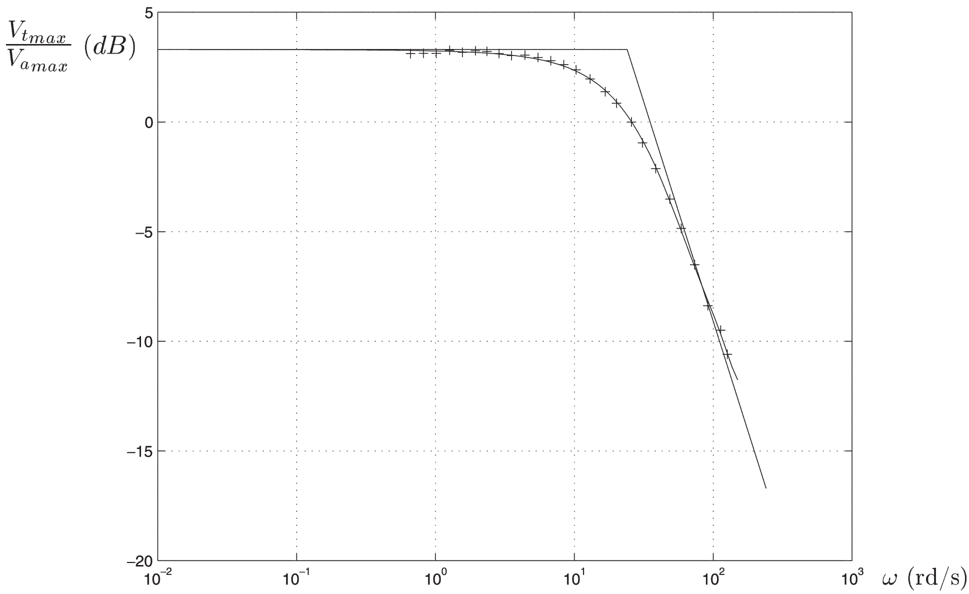

Fig. 9 *Bode diagram for the d.c. motor-generator group of UFRJ.*

voltage in the tachometer terminals must, in steady-state, remain equal to the reference voltage;

5 Good transient response.

Notice that control objective 2 requires that the reference signal to be used be a voltage step of amplitude V_r (volts), which can be mathematically expressed as follows:

$$v_{t_r}(t) = \begin{cases} V_r, & t \geq 0 \\ 0, & t < 0 \end{cases}. \tag{13}$$

In addition, since the system to be controlled is already stable, this plant provides a good opportunity to highlight the benefits of feedback.[1] For this reason the design of an open-loop control system for the d.c. motor-generator group will be considered first.

An open-loop control system for the d.c. motor-generator group

Let us now consider the block diagram of Fig. 10 where $K(s)$ denotes the controller transfer function to be designed in order to satisfy requirements 1 to 5 above. Notice that the nonlinear block used to consider the effect of the dead zone has been removed from the diagram since the system will be supposed to be operating in the linear region. Moreover, since one of the control objectives is low sensitivity to parameter variation and modeling errors, the controller to be designed will indirectly consider the error introduced by neglecting the dead zone effect from the model.

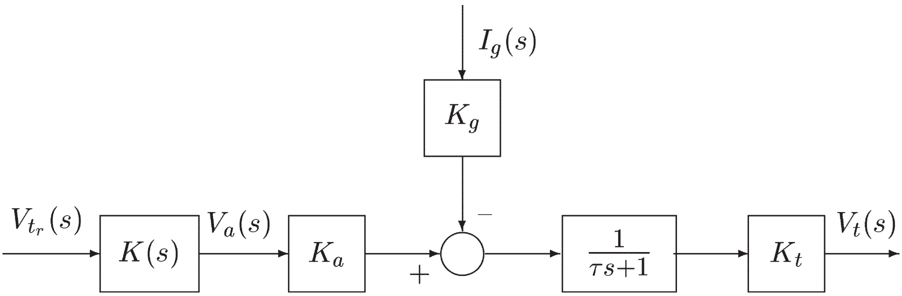

Fig. 10 *Block diagram for the design of an open-loop control system for a d.c. motor-generator group.*

As mentioned before, item 1 (stability) has already been satisfied. In addition since this is a didactic experiment, transient performance specification will be given by the settling time (t_s) of the uncontrolled system. Therefore requirement 4 will be satisfied if, for the compensated system, $t_s \leq 4\tau$.

Therefore, it suffices to use a static compensator,

$$K(s) = K, \tag{14}$$

where K will be determined in order to satisfy requirement 2. It is easy to check that

$$K = \frac{1}{K_a K_t} \tag{15}$$

makes the output signal track the input signal in steady-state providing there is no load connected to the generator terminals.

Once the controller has been designed, the next step is to check the performance of the compensated system. In order to do so, construct a SIMULINK model, equivalent to the block diagram of Fig. 10, using as values of K_a, K_t, K_g and τ those obtained from the identification process and the value of K computed according to (15). Since the open-loop system is already stable and the settling time of the uncompensated plant has been chosen as the measure for good transient response, requirements 1 and 5 have already been met. Thus the main goal of this simulation exercise is to verify whether requirements 2 to 4 are being properly satisfied. In order to do so, the students should proceed as follows:

(i) Suppose, initially, that there is no load connected to the generator terminals i.e. $i_g(t) = 0$ (Ampère) for all t. Next, apply the input (13). The students will be able to verify after the simulation that the zero steady-state error objective was successfully met.

(ii) Assume, now, that there was a 5% error in the identification of the parameter K_a and make this change in the SIMULINK model. Repeat the previous step. This time the students will see that there is a steady-state error of 5%, which shows that the controller has not satisfied objective 3.

(iii) Return the value of K_a in the SIMULINK model to that obtained in the identification. Now excite the system with the reference signal (13) and with a disturbance current given by

$$i_g(t) = \begin{cases} I_g, & t \geq t_0 \\ 0, & t < t_0 \end{cases}. \tag{16}$$

where t_0 is any time instant larger than the settling time. This simulation will show that the open-loop system is not able to reject the disturbance.

A closed-loop control system for the d.c. motor-generator group

Feedback is therefore the proper way to overcome the deficiencies of the open-loop controller. However, before going into the details involved in the controller design it is important to remark that the problem of controlling the shaft velocity of a d.c. motor-generator group also provides an opportunity to illustrate another point, namely the need for a dynamic compensator.[1] In order to do so, consider the feedback system depicted in Fig. 11 and let $G(s) = n_G(s)/d_G(s)$ and $K(s) = n_K(s)/d_K(s)$ denote the plant and controller transfer functions, respectively, where $n_G(s)$ and $d_G(s)$ are known polynomials and $n_K(s)$ and $d_K(s)$ are polynomials to be determined. In addition let $R(s)$, $D(s)$ and $Y(s)$ be the Laplace transforms of the reference, disturbance and output signals. Notice that $Y(s)$ may be written as:

$$Y(s) = Y_R(s) - Y_D(s), \tag{17}$$

where

$$Y_R(s) = \frac{G(s)K(s)}{1 + G(s)K(s)} R(s) \quad \text{and} \quad Y_D(s) = \frac{G(s)}{1 + G(s)K(s)} D(s). \tag{18}$$

Note that the component $Y_R(s)$ accounts for tracking and must be such that $\lim_{t\to\infty} r(t) - y_R(t) = 0$ while the component $Y_D(s)$ is the response to load disturbance and must satisfy $\lim_{t\to\infty} y_D(t) = 0$. The following result may be stated.

Theorem 1 (The internal model principle) Assume that $R(s) = \alpha(s)/\beta(s)$ and $D(s) = \gamma(s)/\delta(s)$ where $\beta(s)$ and $\delta(s)$ are known. Write $\beta(s) = \beta^-(s)\beta^+(s)$ and $\delta(s) = \delta^-(s)\delta^+(s)$ where $\beta^-(s)$ and $\delta^-(s)$ are Hurwitz polynomials and $\beta^+(s)$ and $\delta^+(s)$ has, respectively, all the zeros of $\beta(s)$ and $\delta(s)$ with real part greater than or equal 0. Under the assumption that $K(s)$ stabilizes $G(s)$ then:

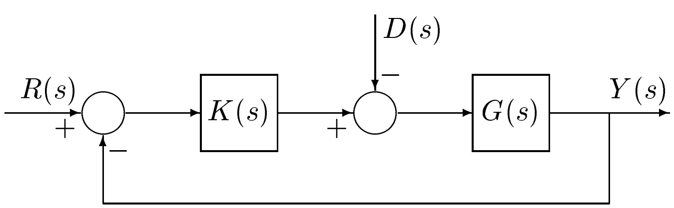

Fig. 11 *Block diagram of a closed-loop system.*

1 The output $y(t)$ will converge asymptotically to $r(t)$, i.e. $\lim_{t\to\infty} r(t) - y_R(t) = 0$ if and only if $d_G(s)d_K(s) = \eta(s)\beta^+(s)$, where $\eta(s)$ is an arbitrary polynomial.
2 The system will reject asymptotically the disturbance signal $d(t)$ i.e. $\lim_{t\to\infty} y_D(t)$ if and only if $n_G(s)d_K(s) = \chi(s)\delta^+(s)$, where $\chi(s)$ is an arbitrary polynomial.

Proof: See Refs. [3] and [7].

From theorem 1 it is possible to conclude that when the reference and disturbance are both step signals then output tracking and disturbance rejection will be achieved provided $d_G(s)d_K(s) = s\eta(s)$ and $n_G(s)d_K(s) = s\chi(s)$, respectively. The only possibility for both conditions be satisfied is $d_K(s) = s\,\bar{d}_K(s)$, where $\bar{d}_K(s)$ is an arbitrary polynomial used to achieve closed-loop stability and 'good' transient performance. Therefore it is necessary to have a dynamic compensator with an integral action.

After this brief review, we may now return to the motor-generator group. Therefore consider the feedback system of Fig. 12 where $K(s)$ is the controller transfer function to be designed. Since $K(s)$ has to be dynamic with an s factor in the denominator, there are several possible controllers, but lower order ones are to be preferred. For this reason the design of a pure integral controller will be considered initially and if necessary $K(s)$ will be changed later.

Design of an integral controller for the d.c. motor-generator group
In this case,

$$K(s) = \frac{K_I}{s} \tag{19}$$

where K_I is to be chosen in order to make the closed-loop system stable with good transient performance.

At this point, another important tool will be used by the students, namely, the root locus. In order to do this notice that the open-loop transfer functions is given by

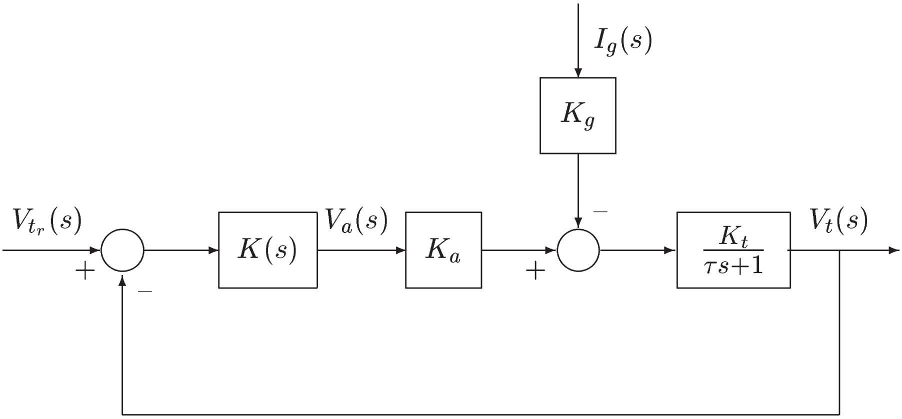

Fig. 12 *Block diagram for the design of a feedback control system of a d.c. motor-generator group.*

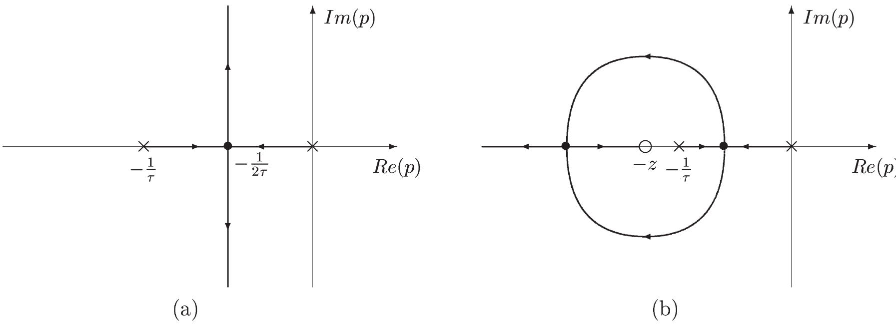

Fig. 13 *Root locus diagram for the design of a controller for the d.c. motor-generator group.*

$$Q(s) = G(s)K(s) = \frac{K_I K_a K_t}{s(\tau s + 1)}, \tag{20}$$

leading to the root locus diagram shown in Fig. 13(a). From the figure it is immediate to see that the closed-loop system is stable for all values of K_I. In addition, it can be seen that when $K_I < 1/(4K_aK_t\tau)$ the closed-loop system will be over-damped, $K_I = 1/(4K_aK_t\tau)$ makes the closed-loop system critically damped while for $K_I > 1/(4K_aK_t\tau)$ the closed-loop response will be under-damped. Another conclusion that can be drawn from the diagram is that even for arbitrary large values of K_I, the real part of the closed-loop poles will be $-1/(2\tau)$ which makes the settling time of the step response for the closed-loop system be approximately 8τ, which is nearly the double of what is required.

Therefore a more complex controller structure, which makes the root locus diagram deviates towards the negative direction of the real axis, is necessary. This can be achieved by a proportional plus integral (PI) controller as will be shown in the sequel.

Design of a PI controller for the d.c. motor-generator group

Notice that by placing a zero $-z$ on the left of the pole $-1/\tau$, as shown in Fig. 13(b) the root locus diagram will actually deviate towards $-z$. This requires $K(s)$ to be as follows:

$$K(s) = \frac{K_P(s+z)}{s} = K_P\left(1+\frac{z}{s}\right) = K_P\left(1+\frac{1}{T_i s}\right), \tag{21}$$

where $T_i = 1/z$, being a PI controller. Therefore the control problem can be stated as follows: find $K_P > 0$ and $z > 0$ ($z > 1/\tau$) such that the closed-loop poles have real part equal $-1/\tau$. It is not hard to check that this is achieved with $K_P = 1/(K_aK_t)$ In addition notice that the choice of z dictates the response overshoot, i.e. the closer to $1/\tau$ the lower the overshoot will be. It is also important to remark that the closed-loop system has now a zero at $-z$ and therefore the settling time may not be approximately equal 4τ. However, this approach serves as a good starting point for the controller design.

Once a PI controller has been designed the students should now build a SIMULINK model equivalent to the block diagram of Fig. 12, using the values of K, K_t, K_g and τ, obtained from the identification experiments, and K_P and z, calculated as above, and carry out simulations to verify that all the design objectives have been satisfied. Another point to be checked is the influence of the position of z in the closed-loop response.

Controller implementation

The last stage of this control laboratory is the controller implementation. Since we are dealing with a first course on control systems, an analog controller will be implemented. The basic structure of a control system is as follows: (i) a sum circuit; (ii) a circuit to perform proportional + integral action; (iii) a power amplifier circuit and (iv) plant (d.c. motor-generator group) with a sensor (tachometer), as shown in Fig. 14. It is important to remark that parts (i) and (ii) can be implemented by means of a very simple circuit if 741 or LF356 operational amplifiers are used, as seen in Fig. 15. The student's final task is to choose appropriately values for R_f, R_i and C_f such that the analog circuit to perform the control action has approximately the same transfer function as that obtained theoretically. In addition, it is important to remark that the power amplifier should be adjusted in order that the real open-loop gain be $K_P K_a K_t$.

Having the controller been implemented, the students will be able to verify that the behavior of the actual compensated system is very close to that obtained in the simulation. The step response of the real compensated system is shown in Fig. 16 (solid lines). The values of R_i, R_f and C_f used in the control circuit are, respectively, $100\,k\Omega$, $35\,k\Omega$ and $0.95\,\mu F$, and the power amplifier has been adjusted to give a gain equal 2, leading to $K_P = 0.7$ and $z \approx 30.1$. In the same figure (dashed line) it is depicted the response obtained from a SIMULINK model by applying the same signal as the one applied to the real system. It can be seen that, as expected, they approximately match each other.

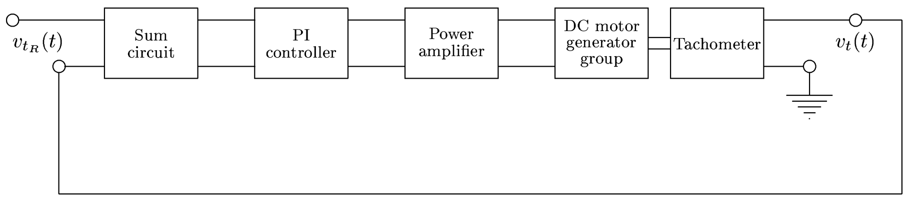

Fig. 14 *Control system for the d.c. motor-generator group.*

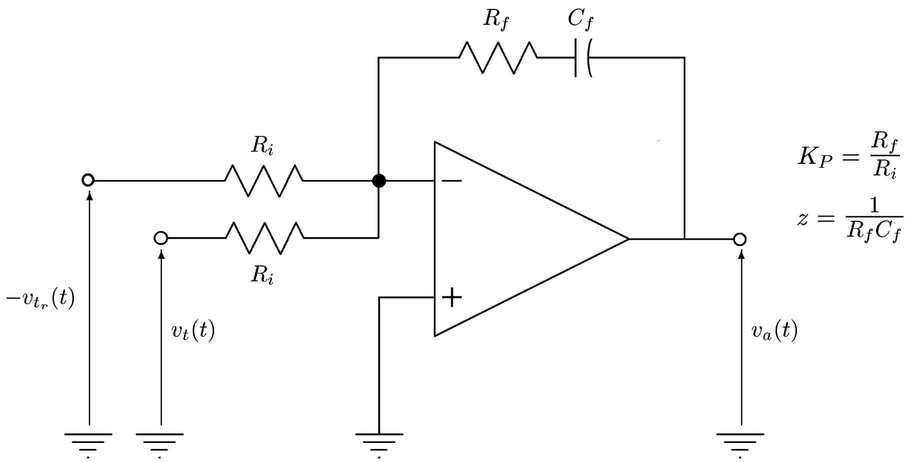

Fig. 15 *Analog circuit for controller implementation.*

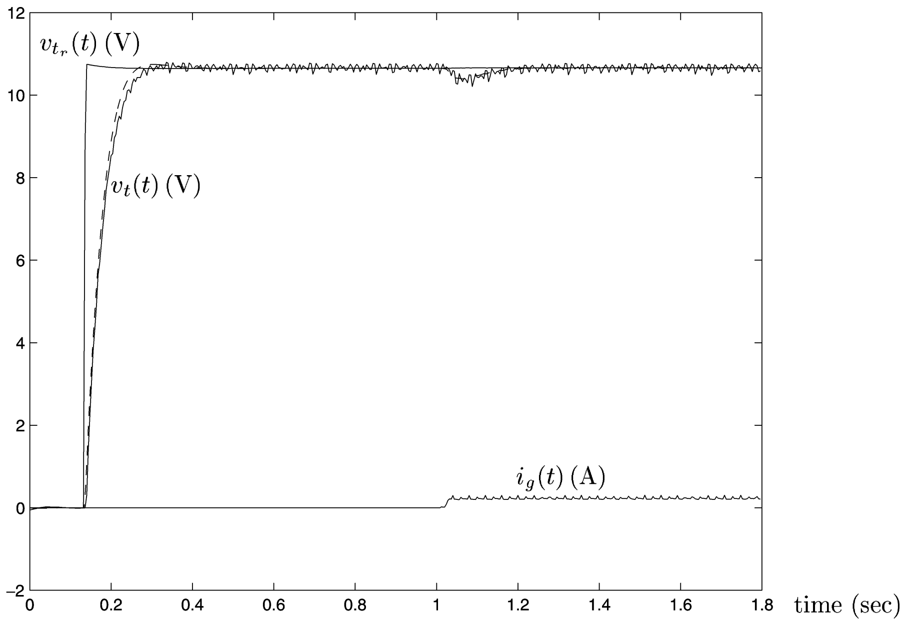

Fig. 16 *Response of the actual feedback system (solid line) and from simulation using the model of Fig. 5 (dashed line) to the same step input and disturbance.*

Conclusions

In this paper, a model for a laboratory for a first course in control systems has been proposed, whose main advantages are: (i) it illustrates all the concepts and tools taught in a first theoretical course in control systems; (ii) it gives the students the opportunity to get in touch with all the stages of the design of a control system and (iii) it is general enough to be used in other plants besides the one deployed in this laboratory.

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